



The CENTRE for EDUCATION  
in MATHEMATICS and COMPUTING  
[cemc.uwaterloo.ca](http://cemc.uwaterloo.ca)

## Gauss Contest

**Grade 7**

(The Grade 8 Contest is on the reverse side)

**Wednesday, May 13, 2015**

(in North America and South America)

**Thursday, May 14, 2015**

(outside of North America and South America)




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Time: 1 hour

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Calculators are allowed, with the following restriction: you may not use a device that has internet access, that can communicate with other devices, or that contains previously stored information. For example, you may not use a smartphone or a tablet.

**Instructions**

1. Do not open the contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your answer sheet. If you are not sure, ask your teacher to explain it.
4. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. When you have made your choice, enter the appropriate letter for that question on your answer sheet.
5. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.  
There is *no penalty* for an incorrect answer.  
Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
6. Diagrams are *not* drawn to scale. They are intended as aids only.
7. When your supervisor instructs you to start, you will have *sixty* minutes of working time.

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*The name, school and location of some top-scoring students will be published on the Web site, [cemc.uwaterloo.ca](http://cemc.uwaterloo.ca). You will also be able to find copies of past Contests and excellent resources for enrichment, problem solving and contest preparation.*

**Grade 7**

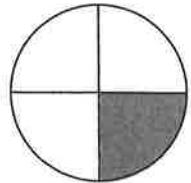
Scoring: There is *no penalty* for an incorrect answer.

Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

**Part A: Each correct answer is worth 5.**

1. In the diagram, the fraction of the circle that is shaded is equal to

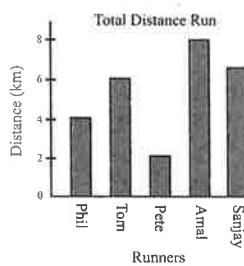
(A)  $\frac{1}{2}$       (B)  $\frac{1}{3}$       (C)  $\frac{1}{4}$   
(D)  $\frac{1}{6}$       (E)  $\frac{1}{8}$



2. The value of  $10 \times (5 - 2)$  is  
(A) 13      (B) 70      (C) 7      (D) 30      (E) 50

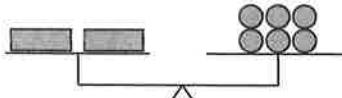
3. The graph shows the total distance that each of five runners ran during a one-hour training session. Which runner ran the least distance?

(A) Phil      (B) Tom      (C) Pete  
(D) Amal      (E) Sanjay



4. The equal-arm scale shown is balanced.  
One has the same mass as

(A)   
(B)   
(C)   
(D)   
(E)



5. Which of the following is closest to 5 cm?

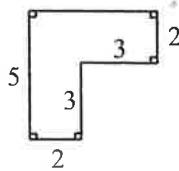
(A) The length of a full size school bus  
(B) The height of a picnic table  
(C) The height of an elephant  
(D) The length of your foot  
(E) The length of your thumb

6. The number of centimetres in 3.5 metres is

(A) 350      (B) 30.5      (C) 3.05      (D) 3.50      (E) 305

7. The perimeter of the figure shown is

(A) 18      (B) 17      (C) 23  
(D) 20      (E) 25



8. Hannah scored 312 points during the basketball season. If her average (mean) was 13 points per game, how many games did she play?

(A) 24      (B) 41      (C) 17      (D) 13      (E) 30

9. The number 6 has exactly four positive divisors: 1, 2, 3, and 6. How many positive divisors does 20 have?

(A) 2      (B) 6      (C) 3      (D) 5      (E) 8

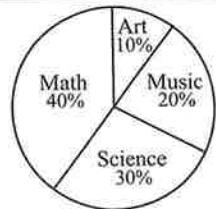
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10. How many different 3-digit whole numbers can be formed using the digits 4, 7 and 9, assuming that no digit can be repeated in a number?
- (A) 6      (B) 3      (C) 5      (D) 12      (E) 9
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**Part B: Each correct answer is worth 6.**

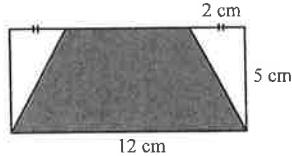
11. At Gaussville School, a total of 480 students voted for their favourite subject. The results are summarized in the pie chart shown. How many students voted for math?
- (A) 184      (B) 192      (C) 96  
 (D) 144      (E) 288



12. A piece of paper is folded in half, creating two layers of paper. The paper is then folded in half again. This is continued until the paper has been folded in half a total of five times. The total number of layers of paper in the folded sheet is
- (A) 16      (B) 32      (C) 25      (D) 8      (E) 64
13. How many even whole numbers between 1 and 99 are multiples of 5?
- (A) 5      (B) 7      (C) 9      (D) 11      (E) 13
14. In the  $3 \times 3$  table shown, the numbers 1, 2 and 3 are placed so that each number occurs only once in each row and only once in each column. The value of  $X + Y$  is
- (A) 3      (B) 2      (C) 5  
 (D) 6      (E) 4

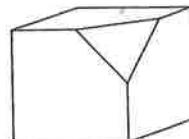
|   |     |  |     |
|---|-----|--|-----|
|   |     |  | 1   |
| 3 | $X$ |  |     |
|   |     |  | $Y$ |

15. In the rectangle shown, the area of the shaded region is
- (A)  $60\text{ cm}^2$       (B)  $20\text{ cm}^2$       (C)  $30\text{ cm}^2$   
 (D)  $40\text{ cm}^2$       (E)  $50\text{ cm}^2$

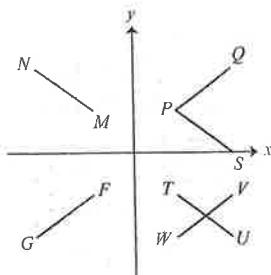


16. You have exactly \$4.40 (440¢) in quarters (25¢ coins), dimes (10¢ coins), and nickels (5¢ coins). You have the same number of each type of coin. How many dimes do you have?
- (A) 20      (B) 11      (C) 10      (D) 12      (E) 4

17. One corner of a cube is cut off, creating a new triangular face, as shown. How many edges does this new solid have?
- (A) 18      (B) 14      (C) 24  
 (D) 15      (E) 13



18. In the graph shown, which of the following represents the image of the line segment  $PQ$  after a reflection across the  $x$ -axis?
- (A)  $PS$       (B)  $TU$       (C)  $MN$   
 (D)  $WV$       (E)  $FG$



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19. When expressed as a repeating decimal, the fraction  $\frac{1}{7}$  is written as 0.142857142857... (The 6 digits 142857 continue to repeat.) The digit in the third position to the right of the decimal point is 2. In which one of the following positions to the right of the decimal point will there also be a 2?
- (A) 119<sup>th</sup>      (B) 121<sup>st</sup>      (C) 123<sup>rd</sup>      (D) 125<sup>th</sup>      (E) 126<sup>th</sup>
20. In a triangle, the measure of one of the angles is  $45^\circ$ . The measures of the other two angles in the triangle are in the ratio 4 : 5. What is the measure of the largest angle in the triangle?
- (A)  $80^\circ$       (B)  $90^\circ$       (C)  $75^\circ$       (D)  $85^\circ$       (E)  $100^\circ$

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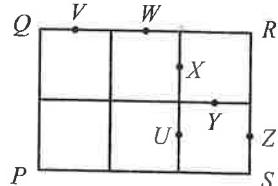
**Part C: Each correct answer is worth 8.**

21. The numbers 1 through 25 are arranged into 5 rows and 5 columns in the table below.

|    |    |    |    |    |
|----|----|----|----|----|
| 1  | 2  | 3  | 4  | 5  |
| 10 | 9  | 8  | 7  | 6  |
| 11 | 12 | 13 | 14 | 15 |
| 20 | 19 | 18 | 17 | 16 |
| 21 | 22 | 23 | 24 | 25 |

What is the largest possible sum that can be made using five of these numbers such that no two numbers come from the same row and no two numbers come from the same column?

- (A) 75      (B) 73      (C) 71      (D) 70      (E) 68
22. The width of a rectangle is doubled and the length is halved. This produces a square with a perimeter of  $P$ . What is the perimeter of the original rectangle?
- (A)  $P$       (B)  $2P$       (C)  $\frac{1}{2}P$       (D)  $\frac{5}{4}P$       (E)  $\frac{5}{2}P$
23. A palindrome is a positive integer that is the same when read forwards or backwards. The numbers 101 and 4554 are examples of palindromes. The ratio of the number of 4-digit palindromes to the number of 5-digit palindromes is
- (A) 4 : 5      (B) 5 : 2      (C) 2 : 7      (D) 4 : 3      (E) 1 : 10
24. In the diagram, rectangle  $PQRS$  is made up of six identical squares. Points  $U, V, W, X, Y$ , and  $Z$  are midpoints of sides of the squares, as shown. Which of the following triangles has the greatest area?
- (A)  $PVU$       (B)  $PXZ$       (C)  $PVX$   
(D)  $PYS$       (E)  $PQW$
25. Two different 2-digit positive integers are called a *reversal pair* if the position of the digits in the first integer is switched in the second integer. For example, 52 and 25 are a reversal pair. The integer 2015 has the property that it is equal to the product of three different prime numbers, two of which are a reversal pair. Including 2015, how many positive integers less than 10 000 have this same property?
- (A) 18      (B) 14      (C) 20      (D) 17      (E) 19



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1. The circle is divided into 4 equal regions. Since 1 of these 4 regions is shaded, then the fraction of the circle that is shaded is  $\frac{1}{4}$ .

ANSWER: (C)

2. Evaluating,  $10 \times (5 - 2) = 10 \times 3 = 30$ .

ANSWER: (D)

3. Reading from the graph, Phil ran 4 km, Tom ran 6 km, Pete ran 2 km, Amal ran 8 km, and Sanjay ran 7 km. Therefore, Pete ran the least distance.

ANSWER: (C)

4. The equal-arm balance shows that 2 rectangles have the same mass as 6 circles.

If we organize these shapes into two equal piles on both sides of the balance, then we see that 1 rectangle has the same mass as 3 circles.

ANSWER: (B)

5. Of the possible answers, the length of your thumb is closest to 5 cm.

ANSWER: (E)

6. There are 100 centimetres in 1 metre. Therefore, there are  $3.5 \times 100 = 350$  cm in 3.5 metres.

ANSWER: (A)

7. The length of the side not labelled is equal to the sum of the two horizontal lengths that are labelled, or  $2 + 3 = 5$ . Thus, the perimeter of the figure shown is  $5 + 5 + 2 + 3 + 3 + 2 = 20$ .

ANSWER: (D)

8. The average (mean) number of points scored per game multiplied by the number of games played is equal to the total number of points scored during the season.

Therefore, the number of games that Hannah played is equal to the total number of points she scored during the season divided by her average (mean) number of points scored per game, or  $312 \div 13 = 24$ .

ANSWER: (A)

9. The positive divisors of 20 are: 1, 2, 4, 5, 10, and 20.

Therefore, the number 20 has exactly 6 positive divisors.

ANSWER: (B)

10. Using the digits 4, 7 and 9 without repeating any digit in a given number, the following 3-digit whole numbers can be formed: 479, 497, 749, 794, 947, and 974.

There are exactly 6 different 3-digit whole numbers that can be formed in the manner described.

ANSWER: (A)

11. *Solution 1*

At Gaussville School, 40% or  $\frac{40}{100} = \frac{4}{10}$  of the 480 total students voted for math.

Therefore, the number of students who voted for math is  $\frac{4}{10} \times 480 = 4 \times 48 = 192$ .

*Solution 2*

At Gaussville School, 40% or 0.4 of the 480 total students voted for math.

Therefore, the number of students who voted for math is  $0.4 \times 480 = 192$ .

ANSWER: (B)

12. The first fold creates 2 layers of paper. The second fold places 2 sets of 2 layers together, for a total of 4 layers of paper. Similarly, the third fold places 2 sets of 4 layers of paper together, for a total of 8 layers of paper.

That is, each new fold places 2 sets of the previous number of layers together, thereby doubling the previous number of layers.

The results of the first five folds are summarized in the table below.

|                  |   |   |   |   |    |    |
|------------------|---|---|---|---|----|----|
| Number of folds  | 0 | 1 | 2 | 3 | 4  | 5  |
| Number of layers | 1 | 2 | 4 | 8 | 16 | 32 |

After the sheet has been folded in half five times, the number of layers in the folded sheet is 32.

ANSWER: (B)

13. *Solution 1*

The multiples of 5 between 1 and 99 are:

$$5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95.$$

Of these, only 10, 20, 30, 40, 50, 60, 70, 80, and 90 are even.

Therefore, there are 9 even whole numbers between 1 and 99 that are multiples of 5.

*Solution 2*

To create an even multiple of 5, we must multiply 5 by an even whole number (since 5 is odd, multiplying 5 by an odd whole number creates an odd result).

The smallest positive even multiple of 5 is  $5 \times 2 = 10$ .

The largest even multiple of 5 less than 99 is  $5 \times 18 = 90$ .

That is, multiplying 5 by each of the even numbers from 2 to 18 results in the only even multiples of 5 between 1 and 99.

Since there are 9 even numbers from 2 to 18 (inclusive), then there are 9 even whole numbers between 1 and 99 that are multiples of 5.

ANSWER: (C)

14. Consider the value of  $U$  in the diagram shown.

Since a 3 already occurs in the second row, then  $U$  cannot equal 3 (each of the numbers 1, 2, 3, occur only once in each row).

Since a 1 already occurs in the third column, then  $U$  cannot equal 1 (each of the numbers 1, 2, 3, occur only once in each column).

Since  $U$  cannot equal 3 or 1, then  $U = 2$ .

Therefore, a 2 and a 3 already occur in the second row and so  $X = 1$ .

At this point, a 2 and a 1 already occur in the third column and so

$Y = 3$ .

The value of  $X + Y = 1 + 3 = 4$ .

ANSWER: (E)

|   |     |     |
|---|-----|-----|
|   |     | 1   |
| 3 | $X$ | $U$ |
|   |     | $Y$ |

15. The rectangle has area  $5 \times 12 = 60 \text{ cm}^2$ .

Each of the two congruent unshaded triangles has area  $\frac{1}{2} \times 2 \times 5 = 5 \text{ cm}^2$ .

The area of the shaded region is equal to the area of the rectangle minus the areas of the two unshaded triangles, which is  $60 - 5 - 5 = 50 \text{ cm}^2$ .

ANSWER: (E)

16. The total value of one quarter, one dime and one nickel is  $25 + 10 + 5 = 40\text{¢}$ .

Since you have equal numbers of quarters, dimes and nickels, you can separate your coins into piles, each containing exactly 1 quarter, 1 dime, and 1 nickel.

Each pile has a value of  $40\text{¢}$ , and since  $440 \div 40 = 11$ , then you must have 11 quarters, 11 dimes and 11 nickels.

Therefore, you have 11 dimes.

Note: You can check that  $11 \times (25\text{¢} + 10\text{¢} + 5\text{¢}) = 11 \times 40\text{¢} = 440\text{¢} = \$4.40$ , as required.

ANSWER: (B)

17. The original cube (before the corner was cut off) had 12 edges.

Cutting off the corner does not eliminate any of the 12 edges of the original cube.

However, cutting off the corner does add 3 edges that were not present originally, the 3 edges of the new triangular face. Since no edges of the original cube were lost, but 3 new edges were created, then the new solid has  $12 + 3 = 15$  edges.

ANSWER: (D)

18. To find the image of  $PQ$ , we reflect points  $P$  and  $Q$  across the  $x$ -axis, then join them.

Since  $P$  is 3 units above the  $x$ -axis, then the reflection of  $P$  across the  $x$ -axis is 3 units below the  $x$ -axis at the same  $x$ -coordinate.

That is, point  $T$  is the image of  $P$  after it is reflected across the  $x$ -axis.

Similarly, after a reflection across the  $x$ -axis, the image of point  $Q$  will be 6 units below the  $x$ -axis but have the same  $x$ -coordinate as  $Q$ .

That is, point  $U$  is the image of  $Q$  after it is reflected across the  $x$ -axis.

Therefore, the line segment  $TU$  is the image of  $PQ$  after it is reflected across the  $x$ -axis.

ANSWER: (B)

19. Since the number of digits that repeat is 6, then the digits 142857 begin to repeat again after 120 digits (since  $120 = 6 \times 20$ ).

That is, the 121<sup>st</sup> digit is a 1, the 122<sup>nd</sup> digit is a 4, and the 123<sup>rd</sup> digit is a 2.

ANSWER: (C)

20. Since the sum of the measures of the three angles in any triangle is  $180^\circ$ , then the sum of the measures of the two unknown angles in the given triangle is  $180^\circ - 45^\circ = 135^\circ$ .

The measures of the two unknown angles are in the ratio 4 : 5, and so one of the two angle measures is  $\frac{5}{4+5} = \frac{5}{9}$  of the sum of the two angles, while the other angle measures  $\frac{4}{4+5} = \frac{4}{9}$  of the sum of the two angles.

That is, the larger of the two unknown angles measures  $\frac{5}{9} \times 135^\circ = 75^\circ$ , and the smaller of the unknown angles measures  $\frac{4}{9} \times 135^\circ = 60^\circ$ .

We may check that  $60^\circ + 75^\circ + 45^\circ = 180^\circ$ .

The largest angle in the triangle measures  $75^\circ$ .

ANSWER: (C)

21. We begin by choosing the largest number in each row, 5, 10, 15, 20, 25, and calling this list  $L$ . The sum of the five numbers in  $L$  is  $5 + 10 + 15 + 20 + 25 = 75$  and this sum satisfies the condition that no two numbers come from the same row.

However, the numbers in  $L$  are taken from columns 1 and 5 only, and the numbers must be chosen so that no two come from the same column.

Thus, the largest of the five answers given, 75, is not possible.

Note: In assuring that we take one number from each row, this choice of numbers,  $L$ , is the only way to obtain a sum of 75 (since we chose the largest number in each row).

Of the five answers given, the next largest answer is 73.

Since  $L$  uses the largest number in each row and has a sum of 75, we can obtain a sum of 73 either by replacing one of the numbers in  $L$  with a number that is two less, or by replacing two of the numbers in  $L$  with numbers that are each one less.

For example, the list 3, 10, 15, 20, 25 (one change to  $L$ ) has sum 73 as does the list 4, 9, 15, 20, 25 (two changes to  $L$ ).

That is, to obtain a sum of 73 while choosing exactly one number from each row, we must choose at least three of the numbers from  $L$ .

However, since two numbers in  $L$  lie in column 1 and three numbers from  $L$  lie in column 5, it is not possible to choose at least three numbers from  $L$  so that no two of the numbers are from the same column.

Any other replacement would give a sum less than 73, which would require the replacement of a number with a larger number in another row to compensate. This is impossible since each row is represented in  $L$  by the largest number in the row.

Therefore, it is not possible to obtain a sum of 73.

Of the five answers given, the next largest answer is 71.

By choosing the numbers, 3, 9, 14, 20, 25 we obtain the sum  $3 + 9 + 14 + 20 + 25 = 71$  while satisfying the condition that no two numbers come from the same row and no two numbers come from the same column.

Thus, 71 is the largest possible sum that satisfies the given conditions.

Note: There are other choices of five numbers which also give a sum of 71 and satisfy the given conditions.

**ANSWER:** (C)

22. Since the perimeter of the square is  $P$  and the 4 sides of a square are equal in length, then each side of the square has length  $\frac{1}{4}P$ . We now work backward to determine the width and length of the rectangle.

The width of the rectangle was doubled to produce the side of the square with length  $\frac{1}{4}P$ .

Therefore, the width of the rectangle is half of the side length of the square, or  $\frac{1}{2} \times \frac{1}{4}P = \frac{1}{8}P$ .

The length of the rectangle was halved to produce the side of the square with length  $\frac{1}{4}P$ .

Therefore, the length of the rectangle is twice the side length of the square, or  $2 \times \frac{1}{4}P = \frac{1}{2}P$ .

Finally, we determine the perimeter of the rectangle having width  $\frac{1}{8}P$  and length  $\frac{1}{2}P$ , obtaining  $2 \times (\frac{1}{8}P + \frac{1}{2}P) = 2 \times (\frac{1}{8}P + \frac{4}{8}P) = 2 \times (\frac{5}{8}P) = \frac{5}{4}P$ .

**ANSWER:** (D)

23. *Solution 1*

Every 4-digit palindrome is of the form  $abba$ , where  $a$  is a digit between 1 and 9 inclusive and  $b$  is a digit between 0 and 9 inclusive (and  $b$  is not necessarily different than  $a$ ).

Every 5-digit palindrome is of the form  $abcba$ , where  $a$  is a digit between 1 and 9 inclusive,  $b$  is a digit between 0 and 9 inclusive (and  $b$  is not necessarily different than  $a$ ), and  $c$  is a digit between 0 and 9 inclusive (and  $c$  is not necessarily different than  $a$  and  $b$ ).

That is, for every 4-digit palindrome  $abba$  there are 10 possible digits  $c$  so that  $abcba$  is a 5-digit palindrome.

For example if  $a = 2$  and  $c = 3$ , then the 4-digit palindrome 2332 can be used to create the 10 5-digit palindromes: 23032, 23132, 23232, 23332, 23432, 23532, 23632, 23732, 23832, 23932.

Thus, for every 4-digit palindrome  $abba$ , there are exactly 10 5-digit palindromes  $abcba$  and so the ratio of the number of 4-digit palindromes to the number of 5-digit palindromes is 1 : 10.

*Solution 2*

Every 4-digit palindrome is of the form  $abba$ , where  $a$  is a digit between 1 and 9 inclusive and  $b$  is a digit between 0 and 9 inclusive (and  $b$  is not necessarily different than  $a$ ).

There are 9 choices for the first digit  $a$  and, for each of these choices, there are 10 choices for the second digit  $b$  or  $9 \times 10 = 90$  choices for the first two digits  $ab$ .

Once the first two digits of the 4-digit palindrome are chosen, then the third and fourth digits are also determined (since the third digit must equal the second and the fourth must equal the first).

That is, there are 90 4-digit palindromes.

Every 5-digit palindrome is of the form  $defed$ , where  $d$  is a digit from 1 to 9 inclusive and  $e$  is a digit from 0 to 9 inclusive (and  $e$  is not necessarily different than  $d$ ) and  $f$  is a digit from 0 to 9 inclusive (and  $f$  is not necessarily different than  $d$  and  $e$ ).

There are 9 choices for the first digit  $d$  and 10 choices for the second digit  $e$  and 10 choices for the third digit  $f$  or  $9 \times 10 \times 10 = 900$  choices for the first three digits  $def$ .

Once the first three digits of the 5-digit palindrome are chosen, then the fourth and fifth digits are also determined (since the fourth digit must equal the second and the fifth must equal the first).

That is, there are 900 5-digit palindromes.

Thus, the ratio of the number of 4-digit palindromes to the number of 5-digit palindromes is  $90 : 900$  or  $1 : 10$ .

**ANSWER:** (E)

24. We can determine which triangle has the greatest area by using a fixed side length of 4 for each of the identical squares and using this to calculate the unknown areas.

We begin by constructing  $\triangle PVU$  and noticing that it is contained within square  $QABP$ , as shown. The area of  $\triangle PVU$  is determined by subtracting the areas of triangles  $PQV$ ,  $VAU$  and  $PBU$  from the area of square  $QABP$ .

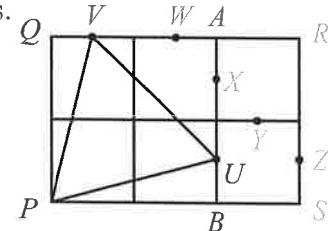
Since  $QA = 8$  and  $AB = 8$ , then the area of square  $QABP$  is  $8 \times 8 = 64$ .

Since  $PQ = 8$  and  $QV = 2$ , then the area of  $\triangle PQV$  is  $\frac{1}{2} \times 8 \times 2 = 8$ .

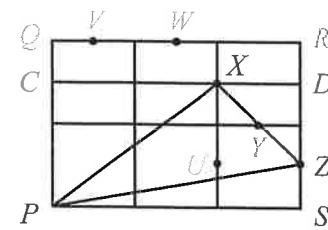
Since  $VA = 6$  and  $AU = 6$ , then the area of  $\triangle VAU$  is  $\frac{1}{2} \times 6 \times 6 = 18$ .

Since  $PB = 8$  and  $UB = 2$ , then the area of  $\triangle PBU$  is  $\frac{1}{2} \times 8 \times 2 = 8$ .

Therefore, the area of  $\triangle PVU$  is  $64 - 8 - 18 - 8 = 30$ .



Next, we construct  $\triangle PXZ$  and then construct rectangle  $CDSP$  by drawing  $CD$  parallel to  $PS$  through  $X$ . Further,  $X$  is the midpoint of the side of a square and so  $C$  and  $D$  are also midpoints of the sides of their respective squares.



The area of  $\triangle PXZ$  is determined by subtracting the areas of triangles  $PCX$ ,  $XDZ$  and  $PSZ$  from the area of rectangle  $CDSP$ .

Since  $CD = 12$  and  $DS = 6$ , then the area of rectangle  $CDSP$  is  $12 \times 6 = 72$ .

Since  $PC = 6$  and  $CX = 8$ , then the area of  $\triangle PCX$  is  $\frac{1}{2} \times 6 \times 8 = 24$ .

Since  $XD = 4$  and  $DZ = 4$ , then the area of  $\triangle XDZ$  is  $\frac{1}{2} \times 4 \times 4 = 8$ .

Since  $PS = 12$  and  $ZS = 2$ , then the area of  $\triangle PSZ$  is  $\frac{1}{2} \times 12 \times 2 = 12$ .

Therefore, the area of  $\triangle PXZ$  is  $72 - 24 - 8 - 12 = 28$ .

Construct  $\triangle PVX$  and notice that it is contained within square  $QABP$ , as shown. The area of  $\triangle PVX$  is determined by subtracting the areas of triangles  $PQV$ ,  $VAX$  and  $PBX$  from the area of square  $QABP$ .

As we previously determined, the area of square  $QABP$  is 64 and the area of  $\triangle PQV$  is 8.

Since  $VA = 6$  and  $AX = 2$ , then the area of  $\triangle VAX$  is  $\frac{1}{2} \times 6 \times 2 = 6$ .

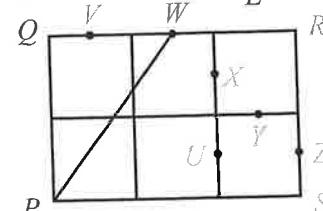
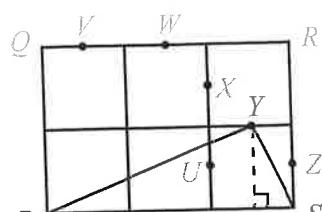
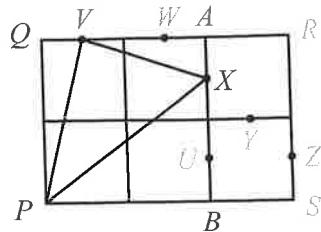
Since  $PB = 8$  and  $XB = 6$ , then the area of  $\triangle PBX$  is  $\frac{1}{2} \times 8 \times 6 = 24$ .

Therefore, the area of  $\triangle PVX$  is  $64 - 8 - 6 - 24 = 26$ .

Construct  $\triangle PYS$  and the perpendicular from  $Y$  to  $PS$ , as shown. Since  $PS = 12$  and  $YE = 4$  ( $YE$  is parallel to  $RS$  and thus equal in length to the side of the square), then the area of  $\triangle PYS$  is  $\frac{1}{2} \times 12 \times 4 = 24$ .

Construct  $\triangle PQW$ , as shown. Since  $PQ = 8$  and  $QW = 6$ , then the area of  $\triangle PQW$  is  $\frac{1}{2} \times 8 \times 6 = 24$ .

The areas of the 5 triangles are 30, 28, 26, 24, and 24. The triangle with greatest area, 30, is  $\triangle PVU$ .



ANSWER: (A)

25. All 2-digit prime numbers are odd numbers, so to create a reversal pair, both digits of each prime must be odd (so that both the original number and its reversal are odd numbers).

We also note that the digit 5 cannot appear in either prime number of the reversal pair since any 2-digit number ending in 5 is not prime.

Combining these two facts together leaves only the following list of prime numbers from which to search for reversal pairs: 11, 13, 17, 19, 31, 37, 71, 73, 79, and 97.

This allows us to determine that the only reversal pairs are: 13 and 31, 17 and 71, 37 and 73, and 79 and 97.

(Note that the reversal of 11 does not produce a different prime number and the reversal of 19 is 91, which is not prime since  $7 \times 13 = 91$ .)

Given a reversal pair, we must determine the prime numbers (different than each prime of the reversal pair) whose product with the reversal pair is a positive integer less than 10 000.

The product of the reversal pair 79 and 97 is  $79 \times 97 = 7663$ .

Since the smallest prime number is 2 and  $2 \times 7663 = 15\,326$ , which is greater than 10 000, then the reversal pair 79 and 97 gives no possibilities that satisfy the given conditions.

We continue in this way, analyzing the other 3 reversal pairs, and summarize our results in the table below.

| Prime Number | Product of the Prime Number with the Reversal Pair |                                |                                |                     |
|--------------|--|--------------------------------|--------------------------------|---------------------|
|              | 13 and 31  | 17 and 71                      | 37 and 73                      | 79 and 97           |
| 2            | $2 \times 13 \times 31 = 806$                      | $2 \times 17 \times 71 = 2414$ | $2 \times 37 \times 73 = 5402$ | greater than 10 000 |
| 3            | $3 \times 13 \times 31 = 1209$                     | $3 \times 17 \times 71 = 3621$ | $3 \times 37 \times 73 = 8103$ |                     |
| 5            | $5 \times 13 \times 31 = 2015$                     | $5 \times 17 \times 71 = 6035$ | greater than 10 000            |                     |
| 7            | $7 \times 13 \times 31 = 2821$                     | $7 \times 17 \times 71 = 8449$ |                                |                     |
| 11           | $11 \times 13 \times 31 = 4433$                    | greater than 10 000            |                                |                     |
| 13           | can't use 13 twice                                 |                                |                                |                     |
| 17           | $17 \times 13 \times 31 = 6851$                    |                                |                                |                     |
| 19           | $19 \times 13 \times 31 = 7657$                    |                                |                                |                     |
| 23           | $23 \times 13 \times 31 = 9269$                    |                                |                                |                     |
| 29           | greater than 10 000                                |                                |                                |                     |
| Total        | 8  | 4                              | 2                              | 0                   |

In any column, once we obtain a product that is greater than 10 000, we may stop evaluating subsequent products since they use a larger prime number and thus will exceed the previous product.

In total, there are  $8 + 4 + 2 = 14$  positive integers less than 10 000 which have the required property.

ANSWER: (B)