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The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
*cemc.uwaterloo.ca*

# *Gauss Contest*

## *Grade 7*

***(The Grade 8 Contest is on the reverse side)***

***Wednesday, May 10, 2017***
***(in North America and South America)***

***Thursday, May 11, 2017***
***(outside of North America and South America)***

UNIVERSITY OF WATERLOO

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**Time:** 1 hour

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**Calculators are allowed, with the following restriction: you may not use a device that has internet access, that can communicate with other devices, or that contains previously stored information. For example, you may not use a smartphone or a tablet.**

**Instructions**

1. Do not open the contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your answer sheet. If you are not sure, ask your teacher to explain it.
4. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. When you have made your choice, enter the appropriate letter for that question on your answer sheet.
5. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
6. Diagrams are *not* drawn to scale. They are intended as aids only.
7. When your supervisor instructs you to start, you will have *sixty* minutes of working time.

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*The name, school and location of some top-scoring students will be published on the Web site, cemc.uwaterloo.ca. You will also be able to find copies of past Contests and excellent resources for enrichment, problem solving and contest preparation.*

## Grade 7

| Scoring: | There is *no penalty* for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions. |
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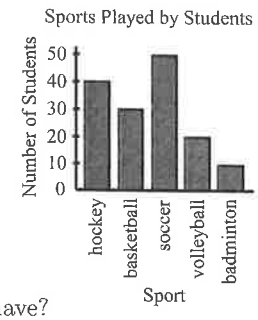
**Part A: Each correct answer is worth 5.**

1. The value of $(2+4+6)-(1+3+5)$ is

   **(A)** 0 **(B)** 3 **(C)** $-3$ **(D)** 21 **(E)** 111

2. Based on the graph shown, which sport is played by the most students?

   **(A)** hockey **(B)** basketball **(C)** soccer **(D)** volleyball **(E)** badminton




3. Michael has \$280 in \$20 bills. How many \$20 bills does he have?

   **(A)** 10 **(B)** 12 **(C)** 14 **(D)** 16 **(E)** 18

4. When two integers between 1 and 10 are multiplied, the result is 14. What is the sum of these two integers?

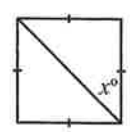
   **(A)** 2 **(B)** 5 **(C)** 7 **(D)** 9 **(E)** 33

5. Three thousandths is equal to

   **(A)** 300 **(B)** 0.3 **(C)** 0.03 **(D)** 30 **(E)** 0.003

6. In the square shown, $x$ is equal to

   **(A)** 0 **(B)** 45 **(C)** 60 **(D)** 180 **(E)** 360




7. Which integer is closest in value to $\frac{35}{4}$?

   **(A)** 10 **(B)** 8 **(C)** 9 **(D)** 7 **(E)** 6

8. When $n = 101$, which of the following expressions has an even value?

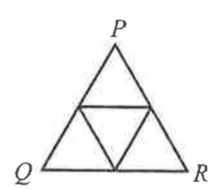
   **(A)** $3n$ **(B)** $n+2$ **(C)** $n-12$ **(D)** $2n-2$ **(E)** $3n+2$

9. The sum of three consecutive integers is 153. The largest of these three integers is

   **(A)** 52 **(B)** 50 **(C)** 53 **(D)** 54 **(E)** 51

10. In the diagram, $\triangle PQR$ is equilateral and is made up of four smaller equilateral triangles. If each of the smaller triangles has a perimeter of 9 cm, what is the perimeter of $\triangle PQR$?

    **(A)** 15 cm **(B)** 9 cm **(C)** 36 cm **(D)** 27 cm **(E)** 18 cm



**Part B: Each correct answer is worth 6.**

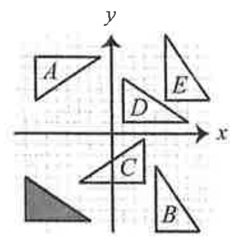
11. The number that goes into the □ to make $\frac{3}{7} = \frac{\square}{63}$ true is

(A) 27 (B) 9 (C) 59 (D) 63 (E) 3

12. At the Gaussian Store, puzzles cost \$10 each or \$50 for a box of 6 puzzles. If a customer would like exactly 25 puzzles, what is the minimum possible cost?

(A) \$210 (B) \$230 (C) \$250 (D) \$220 (E) \$200

13. When the shaded triangle shown is translated, which of the following triangles can be obtained?




(A) $A$ (B) $B$ (C) $C$
(D) $D$ (E) $E$

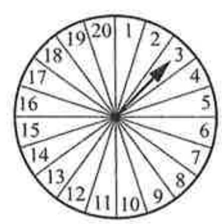
14. When the time in Toronto, ON is 1:00 p.m., the time in Gander, NL is 2:30 p.m. A flight from Toronto to Gander takes 2 hours and 50 minutes. If the flight departs at 3:00 p.m. (Toronto time), what time will the flight land in Gander (Gander time)?

(A) 7:20 p.m. (B) 5:00 p.m. (C) 6:20 p.m. (D) 5:20 p.m. (E) 8:50 p.m.

15. Five students ran a race. Ryan was faster than Henry and Faiz. Henry was slower than Faiz. Toma was faster than Ryan but slower than Omar. Which student finished fourth?

(A) Faiz (B) Henry (C) Omar (D) Ryan (E) Toma

16. A circular spinner is divided into 20 equal sections, as shown. An arrow is attached to the centre of the spinner. The arrow is spun once. What is the probability that the arrow stops in a section containing a number that is a divisor of 20?




(A) $\frac{12}{20}$ (B) $\frac{14}{20}$ (C) $\frac{15}{20}$
(D) $\frac{7}{20}$ (E) $\frac{6}{20}$

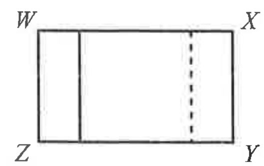
17. The mean (average) of the four integers $78, 83, 82$, and $x$ is 80. Which one of the following statements is true?

(A) $x$ is 2 greater than the mean
(B) $x$ is 1 less than the mean
(C) $x$ is 2 less than the mean
(D) $x$ is 3 less than the mean
(E) $x$ is equal to the mean

18. Sara goes to a bookstore and wants to buy a book that is originally priced at \$100. Which of the following options gives her the best discounted price?

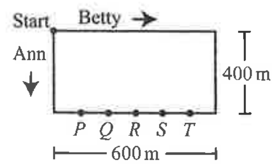
(A) A discount of 20%
(B) A discount of 10%, then a discount of 10% off the new price
(C) A discount of 15%, then a discount of 5% off the new price
(D) A discount of 5%, then a discount of 15% off the new price
(E) All four options above give the same price

19. Two sheets of 11 cm × 8 cm paper are placed on top of each other, forming an overlapping 8 cm × 8 cm square in the centre, as shown. The area of rectangle $WXYZ$ is




(A) $88\,\text{cm}^2$ (B) $112\,\text{cm}^2$ (C) $136\,\text{cm}^2$
(D) $121\,\text{cm}^2$ (E) $176\,\text{cm}^2$

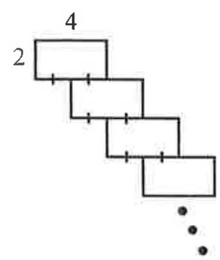
20. Betty and Ann are walking around a rectangular park with dimensions 600 m by 400 m, as shown. They both begin at the top left corner of the park and walk at constant but different speeds. Betty walks in a clockwise direction and Ann walks in a counterclockwise direction. Points $P, Q, R, S, T$ divide the bottom edge of the park into six segments of equal length. When Betty and Ann meet for the first time, they are between $Q$ and $R$. Which of the following could be the ratio of Betty's speed to Ann's speed?




(A) 5 : 3 (B) 9 : 4 (C) 11 : 6
(D) 12 : 5 (E) 17 : 7

## Part C: Each correct answer is worth 8.

21. Rectangles that measure $4 \times 2$ are positioned in a pattern in which the top left vertex of each rectangle (after the top one) is placed at the midpoint of the bottom edge of the rectangle above it, as shown. When a total of ten rectangles are arranged in this pattern, what is the perimeter of the figure?




(A) 48 (B) 64 (C) 90
(D) 84 (E) 100

22. In the six-digit number $1ABCDE$, each letter represents a digit. Given that $1ABCDE \times 3 = ABCDE1$, the value of $A + B + C + D + E$ is

(A) 29 (B) 26 (C) 22 (D) 30 (E) 28

23. Given 8 dimes (10¢ coins) and 3 quarters (25¢ coins), how many different amounts of money can be created using one or more of the 11 coins?

(A) 27 (B) 29 (C) 35 (D) 26 (E) 28

24. Four vertices of a quadrilateral are located at $(7, 6)$, $(-5, 1)$, $(-2, -3)$, and $(10, 2)$. The area of the quadrilateral in square units is

(A) 60 (B) 63 (C) 67 (D) 70 (E) 72

25. Ashley writes out the first 2017 positive integers. She then underlines any of the 2017 integers that is a multiple of 2, and then underlines any of the 2017 integers that is a multiple of 3, and then underlines any of the 2017 integers that is a multiple of 5. Finally, Ashley finds the sum of all the integers which have *not* been underlined. What is this sum?

(A) 542 708 (B) 543 213 (C) 542 203 (D) 543 326 (E) 543 618

## Grade 7

1. Evaluating, $(2+4+6)-(1+3+5)=12-9=3$.

ANSWER: (B)

2. Reading from the tallest bar on the graph, approximately 50 students play soccer.
Since this is larger than the number of students who play any of the other sports, then soccer is played by the most students.

ANSWER: (C)

3. Michael has \$280 in \$20 bills and so the number of \$20 bills that he has is $280 \div 20 = 14$.

ANSWER: (C)

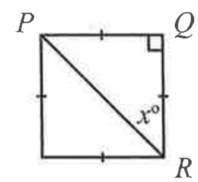
4. There are only two different products of two positive integers whose result is 14.
These are $2 \times 7$ and $1 \times 14$.
Since the two integers must be between 1 and 10, then the product must be $2 \times 7$.
The sum of these two integers is $2+7=9$.

ANSWER: (D)

5. Written as a fraction, three thousandths is equal to $\frac{3}{1000}$.
As a decimal, three thousandths is equal to $3 \div 1000 = 0.003$.

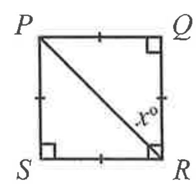
ANSWER: (E)

6. *Solution 1*




Since the given figure is a square, then $PQ = QR$ and $\angle PQR = 90^\circ$.
Since $PQ = QR$, $\triangle PQR$ is isosceles and so $\angle QPR = \angle QRP = x^\circ$.
The three angles in any triangle add to $180^\circ$ and since $\angle PQR = 90^\circ$, then $\angle QPR + \angle QRP = 180^\circ - 90^\circ = 90^\circ$.
Since $\angle QPR = \angle QRP$, then $\angle QRP = 90^\circ \div 2 = 45^\circ$, and so $x = 45$.

*Solution 2*




Diagonal $PR$ divides square $PQRS$ into two identical triangles: $\triangle PQR$ and $\triangle PSR$.
Since these triangles are identical, $\angle PRS = \angle PRQ = x^\circ$.
Since $PQRS$ is a square, then $\angle QRS = 90^\circ$.
That is, $\angle PRS + \angle PRQ = 90^\circ$ or $x^\circ + x^\circ = 90^\circ$ or $2x = 90$ and so $x = 45$.

ANSWER: (B)

7. *Solution 1*
Written as a mixed fraction, $\frac{35}{4} = 8\frac{3}{4}$.
Since $\frac{3}{4}$ is closer to 1 than it is to 0, then $8\frac{3}{4}$ is closer to 9 than it is to 8.
The integer closest in value to $\frac{35}{4}$ is 9.

*Solution 2*
Written as a decimal $\frac{35}{4} = 35 \div 4 = 8.75$.
Since 0.75 is closer to 1 than it is to 0, then 8.75 is closer to 9 than it is to 8.
The integer closest in value to $\frac{35}{4}$ is 9.

ANSWER: (C)

8. When $n = 101$:

$$\begin{aligned} 3n &= 3 \times 101 = 303 \\ n+2 &= 101 + 2 = 103 \\ n-12 &= 101 - 12 = 89 \\ 2n-2 &= 2 \times 101 - 2 = 202 - 2 = 200 \\ 3n+2 &= 3 \times 101 + 2 = 303 + 2 = 305. \end{aligned}$$

Of the expressions given, $2n-2$ is the only expression which has an even value when $n = 101$. (In fact, the value of $2n-2$ is an even integer for every integer $n$. Can you see why?)

ANSWER: (D)

9. The mean (average) of three integers whose sum is 153 is $\frac{153}{3} = 51$.
The mean of three consecutive integers equals the middle of the three integers.
That is, 51 is the middle integer of three consecutive integers and so the largest of these integers is 52.
(We may check that $50 + 51 + 52 = 153$.)

ANSWER: (A)

10. Each of the 4 smaller triangles is equilateral and thus has sides of equal length.
Each of these smaller triangles has a perimeter of 9 cm and so has sides of length $\frac{9}{3} = 3$ cm.
In $\triangle PQR$, side $PQ$ is made up of two such sides of length 3 cm and thus $PQ = 2 \times 3 = 6$ cm.
Since $\triangle PQR$ is equilateral, then $PR = QR = PQ = 6$ cm.
Therefore, the perimeter of $\triangle PQR$ is $3 \times 6 = 18$ cm.

ANSWER: (E)

11. The denominators of the two fractions are 7 and 63.
Since $7 \times 9 = 63$, then we must also multiply the numerator 3 by 9 so that the fractions are equivalent.
That is, $\dfrac{3}{7} = \dfrac{3 \times 9}{7 \times 9} = \dfrac{27}{63}$.
Therefore, the number that goes into the $\square$ so that the statement true is 27.

ANSWER: (A)

12. If puzzles are bought individually for \$10 each, then 6 puzzles will cost $\$10 \times 6 = \$60$.
Since the cost for a box of 6 puzzles is \$50, it is less expensive to buy puzzles by the box than it is to buy them individually.
Buying 4 boxes of 6 puzzles gives the customer $4 \times 6 = 24$ puzzles and the cost is $4 \times \$50 = \$200$.
Buying one additional puzzle for \$10 gives the customer 25 puzzles at a minimum cost of \$210.

ANSWER: (A)

13. A translation moves (slides) an object some distance without altering it in any other way.
That is, the object is not rotated, reflected, and its exact size and shape are maintained.
Of the triangles given, the triangle labelled $D$ is the only triangle whose orientation is identical to that of the shaded triangle.
Thus, $D$ is the triangle which can be obtained when the shaded triangle is translated.

ANSWER: (D)

14. Since the time in Toronto, ON is 1:00 p.m. when the time in Gander, NL is 2:30 p.m., then the time in Gander is 1 hour and 30 minutes ahead of the time in Toronto.
A flight that departs from Toronto at 3:00 p.m. and takes 2 hours and 50 minutes will land in Gander at 5:50 p.m. Toronto time.
When the time in Toronto is 5:50 p.m., the time in Gander is 1 hour and 30 minutes ahead which is 7:20 p.m.

ANSWER: (A)

15. Henry was slower than Faiz and thus finished the race behind Faiz.
Ryan was faster than Henry and Faiz and thus finished the race ahead of both of them.
From fastest to slowest, these three runners finished in the order Ryan, Faiz and then Henry.
Toma was faster than Ryan but slower than Omar.
Therefore, from fastest to slowest, the runners finished in the order Omar, Toma, Ryan, Faiz, and Henry.
The student who finished fourth was Faiz.

ANSWER: (A)

16. The positive divisors of 20 are: $1, 2, 4, 5, 10, 20$.
Of the 20 numbers on the spinner, 6 of the numbers are divisors of 20.
It is equally likely that the spinner lands on any of the 20 numbers.
Therefore, the probability that the spinner lands on a number that is a divisor of 20 is $\frac{6}{20}$.

ANSWER: (E)

17. *Solution 1*
Since 78 is 2 less than 80 and 82 is 2 greater than 80, the mean of 78 and 82 is 80.
Since the mean of all four integers is 80, then the mean of 83 and $x$ must also equal 80.
The integer 83 is 3 greater than 80, and so $x$ must be 3 less than 80.
That is, $x = 80 - 3 = 77$.
(We may check that the mean of $78, 83, 82$, and 77 is indeed 80.)

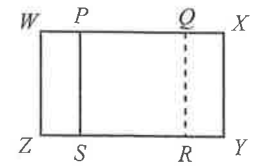
*Solution 2*
Since the mean of the four integers is 80, then the sum of the four integers is $4 \times 80 = 320$.
Since the sum of 78, 83 and 82 is 243, then $x = 320 - 243 = 77$.
Therefore, $x$ is 77 which is 3 less than the mean 80.

ANSWER: (D)

18. A discount of 20% on a book priced at \$100 is a $0.20 \times \$100 = \$20$ discount.
Thus for option (A), Sara's discounted price is $\$100 - \$20 = \$80$.
A discount of 10% on a book priced at \$100 is a $0.10 \times \$100 = \$10$ discount, giving a discounted price of $\$100 - \$10 = \$90$.
A second discount of 10% on the new price of \$90 is a $0.10 \times \$90 = \$9$ discount.
Thus for option (B), Sara's discounted price is $\$90 - \$9 = \$81$.
A discount of 15% on a book priced at \$100 is a $0.15 \times \$100 = \$15$ discount, giving a discounted price of $\$100 - \$15 = \$85$.
A further discount of 5% on the new price of \$85 is a $0.05 \times \$85 = \$4.25$ discount.
Thus for option (C), Sara's discounted price is $\$85 - \$4.25 = \$80.75$.
A discount of 5% on a book priced at \$100 is a $0.05 \times \$100 = \$5$ discount, giving a discounted price of $\$100 - \$5 = \$95$.
A further discount of 15% on the new price of \$95 is a $0.15 \times \$95 = \$14.25$ discount.
Thus for option (D), Sara's discounted price is $\$95 - \$14.25 = \$80.75$.
Therefore, the four options do not give the same price and option (A) gives Sara the best discounted price.

ANSWER: (A)

19. In the diagram, rectangles $WQRZ$ and $PXYS$ are the two sheets of 11 cm$\times$ 8 cm paper.
The overlapping square $PQRS$ has sides of length 8 cm.
That is, $WQ = ZR = PX = SY = 11$ cm and
$WZ = QR = PS = XY = PQ = SR = 8$ cm.
In rectangle $WQRZ$, $WQ = WP + PQ = 11$ cm and so
$WP = 11 - PQ = 11 - 8 = 3$ cm.
In rectangle $WXYZ$, $WX = WP + PX = 3 + 11 = 14$ cm.
Since $WX = 14$ cm and $XY = 8$ cm, the area of $WXYZ$ is
$14 \times 8 = 112$ cm$^2$.




ANSWER: (B)

20. Points $P, Q, R, S, T$ divide the bottom edge of the park into six segments of equal length, each of which has length $600 \div 6 = 100$ m.
If Betty and Ann had met for the first time at point $Q$, then Betty would have walked a total distance of $600 + 400 + 4 \times 100 = 1400$ m and Ann would have walked a total distance of $400 + 2 \times 100 = 600$ m.
When they meet, the time that Betty has been walking is equal to the time that Ann has been walking and so the ratio of Betty's speed to Ann's speed is equal to the ratio of the distance that Betty has walked to the distance that Ann has walked.
That is, if they had met for the first time at point $Q$, the ratio of their speed's would be $1400 : 600$ or $14 : 6$ or $7 : 3$.
Similarly, if Betty and Ann had met for the first time at point $R$, then Betty would have walked a total distance of $600 + 400 + 3 \times 100 = 1300$ m and Ann would have walked a total distance of $400 + 3 \times 100 = 700$ m.
In this case, the ratio of their speed's would be $1300 : 700$ or $13 : 7$.
When Betty and Ann actually meet for the first time, they are between $Q$ and $R$.
Thus Betty has walked less distance than she would have had they met at $Q$ and more distance than she would have had they met at $R$.
That is, the ratio of Betty's speed to Ann's speed must be less than $7 : 3$ and greater than $13 : 7$.
We must determine which of the five given answers is a ratio that is less than $7 : 3$ and greater than $13 : 7$.
One way to do this is to convert each ratio into a mixed fraction.
That is, we must determine which of the five answers is less than $7 : 3 = \frac{7}{3} = 2\frac{1}{3}$ and greater than $13 : 7 = \frac{13}{7} = 1\frac{6}{7}$.
Converting the answers, we get $\frac{5}{3} = 1\frac{2}{3}, \frac{9}{4} = 2\frac{1}{4}, \frac{11}{6} = 1\frac{5}{6}, \frac{12}{5} = 2\frac{2}{5}$, and $\frac{17}{7} = 2\frac{3}{7}$.
Of the five given answers, the only fraction that is less than $2\frac{1}{3}$ and greater than $1\frac{6}{7}$ is $2\frac{1}{4}$.
If Betty and Ann meet for the first time between $Q$ and $R$, then the ratio of Betty's speed to Ann's speed could be $9 : 4$.

ANSWER: (B)

21. *Solution 1*
The first and tenth rectangles each contribute an equal amount to the perimeter.
They each contribute two vertical sides (each of length 2), one full side of length 4 (the top side for the first rectangle and the bottom side for the tenth rectangle), and one half of the length of the opposite side.
That is, the first and tenth rectangles each contribute $2 + 2 + 4 + 2 = 10$ to the perimeter.
Rectangles two through nine each contribute an equal amount to the perimeter.

They each contribute two vertical sides (each of length 2), one half of a side of length 4, and one half of the length of the opposite side (which also has length 4).
That is, rectangles two through nine each contribute $2 + 2 + 2 + 2 = 8$ to the perimeter.
Therefore, the total perimeter of the given figure is $(2 \times 10) + (8 \times 8) = 20 + 64 = 84$.

*Solution 2*
One method for determining the perimeter of the given figure is to consider vertical lengths and horizontal lengths.
Each of the ten rectangles has two vertical sides (a left side and a right side) which contribute to the perimeter.
These 20 sides each have length 2, and thus contribute $20 \times 2 = 40$ to the perimeter of the figure.
Since these are the only vertical lengths contributing to the perimeter, we now determine the sum of the horizontal lengths.

There are two types of horizontal lengths which contribute to the perimeter: the bottom side of a rectangle, and the top side of a rectangle.
The bottom side of each of the first nine rectangles contributes one half of its length to the perimeter.
That is, the bottom sides of the first nine rectangles contribute $\frac{1}{2} \times 4 \times 9 = 18$ to the perimeter.
The entire bottom side of the tenth rectangle is included in the perimeter and thus contributes a length of 4.
Similarly, the top sides of the second rectangle through to the tenth rectangle contribute one half of their length to the perimeter.
That is, the top sides of rectangles two through ten contribute $\frac{1}{2} \times 4 \times 9 = 18$ to the perimeter.
The entire top side of the first rectangle is included in the perimeter and thus contributes a length of 4.
In total, the horizontal lengths included in the perimeter sum to $18 + 4 + 18 + 4 = 44$.
Since there are no additional lengths which contribute to the perimeter of the given figure, the total perimeter is $40 + 44 = 84$.

*Solution 3*
Before they were positioned to form the given figure, each of the ten rectangles had a perimeter of $2 \times (2 + 4) = 12$.
When the figure was formed, some length of each of the ten rectangles' perimeter was "lost" (and thus is not included) in the perimeter of the given figure.
These lengths that were lost occur where the rectangles touch one another.
There are nine such locations where two rectangles touch one another (between the first and second rectangle, between the second and third rectangle, and so on).
In these locations, each of the two rectangles has one half of a side of length 4 which is not included in the perimeter of the given figure.
That is, the portion of the total perimeter of the ten rectangles that is not included in the perimeter of the figure is $9 \times (2 + 2) = 36$.
Since the total perimeter of the ten rectangles before they were positioned into the given figure is $10 \times 12 = 120$, then the perimeter of the given figure is $120 - 36 = 84$.

ANSWER: (D)

22. The units digit of the product $1ABCDE \times 3$ is 1, and so the units digit of $E \times 3$ must equal 1.
Therefore, the only possible value of $E$ is 7.
Substituting $E = 7$, we get

$$\begin{array}{r} 1\,A\,B\,C\,D\,7 \\ \times \qquad\quad 3 \\ \hline A\,B\,C\,D\,7\,1 \end{array}$$

Since $7 \times 3 = 21$, 2 is carried to the tens column.
Thus, the units digit of $D \times 3 + 2$ is 7, and so the units digit of $D \times 3$ is 5.
Therefore, the only possible value of $D$ is 5.
Substituting $D = 5$, we get

$$\begin{array}{r} 1\,A\,B\,C\,5\,7 \\ \times \qquad\quad 3 \\ \hline A\,B\,C\,5\,7\,1 \end{array}$$

Since $5 \times 3 = 15$, 1 is carried to the hundreds column.
Thus, the units digit of $C \times 3 + 1$ is 5, and so the units digit of $C \times 3$ is 4.
Therefore, the only possible value of $C$ is 8.
Substituting $C = 8$, we get

$$\begin{array}{r} 1\,A\,B\,8\,5\,7 \\ \times \qquad\quad 3 \\ \hline A\,B\,8\,5\,7\,1 \end{array}$$

Since $8 \times 3 = 24$, 2 is carried to the thousands column.
Thus, the units digit of $B \times 3 + 2$ is 8, and so the units digit of $B \times 3$ is 6.
Therefore, the only possible value of $B$ is 2.
Substituting $B = 2$, we get

$$\begin{array}{r} 1\,A\,2\,8\,5\,7 \\ \times \qquad\quad 3 \\ \hline A\,2\,8\,5\,7\,1 \end{array}$$

Since $2 \times 3 = 6$, there is no carry to the ten thousands column.
Thus, the units digit of $A \times 3$ is 2.
Therefore, the only possible value of $A$ is 4.
Substituting $A = 4$, we get

$$\begin{array}{r} 1\,4\,2\,8\,5\,7 \\ \times \qquad\quad 3 \\ \hline 4\,2\,8\,5\,7\,1 \end{array}$$

Checking, we see that the product is correct and so $A+B+C+D+E = 4+2+8+5+7 = 26$.

ANSWER: (B)

23. Given 8 dimes (10 ¢ coins) and 3 quarters (25 ¢ coins), we list the different amounts of money (in cents) that can be created in the table below.
When an amount of money already appears in the table, it has been stroked out.

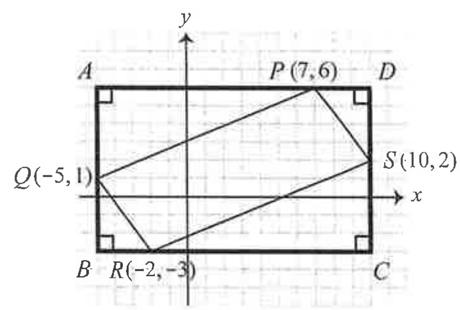
**Number of Dimes**

| Number of Quarters: 25 ¢ \ 10 ¢ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 |
| 1 | 25 | 35 | 45 | 55 | 65 | 75 | 85 | 95 | 105 |
| 2 | ~~50~~ | ~~60~~ | ~~70~~ | ~~80~~ | 90 | 100 | 110 | 120 | 130 |
| 3 | ~~75~~ | ~~85~~ | ~~95~~ | ~~105~~ | 115 | 125 | 135 | 145 | 155 |

We may ignore the first entry in the table, 0, since we are required to use at least one of the 11 coins.
We are left with 27 different amounts of money that can be created using one or more of the 8 dimes and 3 quarters.

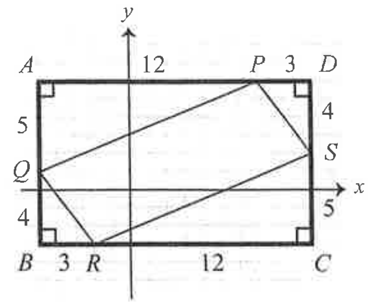
Answer: (A)

24. We begin by constructing rectangle $ABCD$ around the given quadrilateral $PQRS$, as shown.
The vertical sides $AB$ and $DC$ pass through points $Q$ and $S$, respectively.
The horizontal sides $AD$ and $BC$ pass through points $P$ and $R$, respectively.
We determine the area of $PQRS$ by subtracting the areas of the four right-angled triangles, $AQP$, $QBR$, $CSR$, and $SDP$, from the area of $ABCD$.




To determine the horizontal side lengths of the right-angled triangles we count units along the $x$-axis, or we subtract the $x$-coordinates of two vertices.
For example, since $AB$ is vertical and passes through $Q(-5,1)$, the $x$-coordinates of $A$ and $B$ are equal to that of $Q$, which is $-5$.
Thus, the length of $AP$ is determined by subtracting the $x$-coodinate of $A$ from the $x$-coordinate of $P$, which is 7.
Therefore the length of $AP$ is $7-(-5)=12$.
Similarly, the length of $BR$ is $-2-(-5)=3$.
Since $DC$ is vertical and passes through $S(10,2)$, the $x$-coordinates of $D$ and $C$ are equal to that of $S$, which is 10.
Thus, the length of $PD$ is $10-7=3$, and the length of $RC$ is $10-(-2)=12$.
To determine the vertical side lengths of the right-angled triangles we may count units along the $y$-axis, or we may subtract the $y$-coordinates of two vertices.
For example, since $AD$ is horizontal and passes through $P(7,6)$, the $y$-coordinates of $A$ and $D$ are equal to that of $P$, which is 6.
Thus, the length of $AQ$ is determined by subtracting the $y$-coodinate of $Q$ (which is 1) from the $y$-coordinate of $A$.
Therefore the length of $AQ$ is $6-1=5$.
Similarly, the length of $DS$ is $6-2=4$.

Since $BC$ is horizontal and passes through $R(-2,-3)$, the $y$-coordinates of $B$ and $C$ are equal to that of $R$, which is $-3$. Thus, the length of $QB$ is $1-(-3)=4$, and the length of $SC$ is $2-(-3)=5$.




The area of $\triangle AQP$ is $\frac{1}{2}\times AQ\times AP=\frac{1}{2}\times 5\times 12=30$.
The area of $\triangle CSR$ is also 30.
The area of $\triangle QBR$ is $\frac{1}{2}\times QB\times BR=\frac{1}{2}\times 4\times 3=6$.
The area of $\triangle SDP$ is also 6.
Since $AB=AQ+QB=5+4=9$ and $BC=BR+RC=3+12=15$, the area of $ABCD$ is $9\times 15=135$.
Finally, the area of $PQRS$ is $135-30\times 2-6\times 2=135-60-12=63$.

ANSWER: (B)

25. *Solution 1*

The sum of the positive integers from 1 to $n$ is given by the expression $\frac{n(n+1)}{2}$.
For example when $n=6$, the sum $1+2+3+4+5+6$ can be determined by adding these integers to get 21, or by using the expression $\frac{6(6+1)}{2}=\frac{42}{2}=21$.
Using this expression, the sum of the positive integers from 1 to 2017, or $1+2+3+4+\cdots+2016+2017$ is $\frac{2017(2018)}{2}=\frac{4\,070\,306}{2}=2\,035\,153$.
To determine the sum of the integers which Ashley has not underlined, we must subtract from 2 035 153 any of the 2017 integers which is a multiple of 2, or a multiple of 3, or a multiple of 5, while taking care not to subtract any number more than once.
First, we find the sum of all of the 2017 numbers which are a multiple of 2.
This sum contains 1008 integers and is equal to $2+4+6+8+\cdots+2014+2016$.
Since each number in this sum is a multiple of 2, then this sum is equal to twice the sum $1+2+3+4+\cdots+1007+1008$, since $2\times 1=2$, $2\times 2=4$, $2\times 3=6$, and so on.
That is, $2+4+6+8+\cdots+2014+2016=2(1+2+3+4+\cdots+1007+1008)$.
Using the formula above, the sum of the first 1008 positive integers is equal to $\frac{1008(1009)}{2}=\frac{1\,017\,072}{2}=508\,536$, and so
$2+4+6+8+\cdots+2014+2016=2\times 508\,536=1\,017\,072$.
We may similarly determine the sum of all of the 2017 numbers which are a multiple of 3.
This sum is equal to $3+6+9+12+\cdots+2013+2016$ and contains 672 integers (since $3\times 672=2016$).
Since each of these numbers is a multiple of 3, $3+6+9+12+\cdots+2013+2016$ is equal to $3(1+2+3+4+\cdots+671+672)=3\times\frac{672(673)}{2}=3\times\frac{452\,256}{2}=3\times 226\,128=678\,384$.
The sum of all of the 2017 numbers which are a multiple of 5 is equal to
$5+10+15+20+\cdots+2010+2015=5(1+2+3+4+\cdots+402+403)=5\times\frac{403(404)}{2}$ or $5\times 81\,406$ which is equal to 407 030.
We summarize this work in the table below.

| Description | Sum | Result |
|---|---|---|
| All integers from 1 to 2017 | $1+2+3+4+\cdots+2016+2017$ | 2 035 153 |
| Integers that are a multiple of 2 | $2+4+6+8+\cdots+2014+2016$ | 1 017 072 |
| Integers that are a multiple of 3 | $3+6+9+12+\cdots+2013+2016$ | 678 384 |
| Integers that are a multiple of 5 | $5+10+15+20+\cdots+2010+2015$ | 407 030 |

If we now subtract the sum of any of the 2017 integers which is a multiple of 2, or a multiple of 3, or a multiple of 5 from the sum of all 2017 integers, is the result our required sum?
The answer is no. Why?
There is overlap between the list of numbers that are a multiple of 2 and those that are a multiple of 3, and those that are a multiple of 5.
For example, any number that is a multiple of both 2 and 3 (and thus a multiple of 6) has been included in both lists and therefore has been counted twice in our work above.
We must add back into our sum those numbers that are a multiple of 6 (multiple of both 2 and 3), those that are a multiple of 10 (multiple of both 2 and 5), and those that are a multiple of 15 (multiple of both 3 and 5).
The sum of all of the 2017 numbers which are a multiple of 6 is equal to
$6 + 12 + 18 + 24 + \cdots + 2010 + 2016 = 6(1 + 2 + 3 + 4 + \cdots + 335 + 336)$, which is equal to $6\left(\frac{336(337)}{2}\right) = 6 \times 56\,616 = 339\,696$.
The sum of all of the 2017 numbers which are a multiple of 10 is equal to $10 + 20 + 30 + 40 + \cdots + 2000 + 2010 = 10(1 + 2 + 3 + 4 + \cdots + 200 + 201)$, which is equal to $10\left(\frac{201(202)}{2}\right) = 10 \times 20\,301 = 203\,010$.
The sum of all of the 2017 numbers which are a multiple of 15 is equal to $15 + 30 + 45 + 60 + \cdots + 1995 + 2010 = 15(1 + 2 + 3 + 4 + \cdots + 133 + 134)$, which is equal to $15\left(\frac{134(135)}{2}\right) = 15 \times 9045 = 135\,675$.
We again summarize this work in the table below.

| Description | Sum | Result |
|---|---|---|
| All integers from 1 to 2017 | $1 + 2 + 3 + 4 + \cdots + 2016 + 2017$ | 2 035 153 |
| Integers that are a multiple of 2 | $2 + 4 + 6 + 8 + \cdots + 2014 + 2016$ | 1 017 072 |
| Integers that are a multiple of 3 | $3 + 6 + 9 + 12 + \cdots + 2013 + 2016$ | 678 384 |
| Integers that are a multiple of 5 | $5 + 10 + 15 + 20 + \cdots + 2010 + 2015$ | 407 030 |
| Integers that are a multiple of 6 | $6 + 12 + 18 + 24 + \cdots + 2010 + 2016$ | 339 696 |
| Integers that are a multiple of 10 | $10 + 20 + 30 + 40 + \cdots + 2000 + 2010$ | 203 010 |
| Integers that are a multiple of 15 | $15 + 30 + 45 + 60 + \cdots + 1995 + 2010$ | 135 675 |

If we take the sum of all 2017 integers, subtract those that are a multiple of 2, and those that are a multiple of 3, and those that are a multiple of 5, and then add those numbers that were subtracted twice (the multiples of 6, the multiples of 10, and the multiples of 15), then we get:

$$2\,035\,153 - 1\,017\,072 - 678\,384 - 407\,030 + 339\,696 + 203\,010 + 135\,675 = 611\,048$$

Is this the required sum?
The answer is still no, but we are close!
Consider any of the 2017 integers that is a multiple of 2, 3 and 5 (that is, a multiple of $2 \times 3 \times 5 = 30$).
Each number that is a multiple of 30 would have been underlined by Ashley, and therefore should not be included in our sum.
Each multiple of 30 was subtracted from the sum three times (once for each of the multiples of 2, 3 and 5), but then added back into our sum three times (once for each of the mutiples of 6, 10 and 15).
Thus, any of the 2017 integers that is a multiple of 30 must still be subtracted from 611 048 to achieve our required sum.

The sum of all of the 2017 numbers which are a multiple of 30 is equal to $30 + 60 + 90 + 120 + \cdots + 1980 + 2010 = 30(1 + 2 + 3 + 4 + \cdots + 66 + 67)$, which is equal to $30\left(\dfrac{67(68)}{2}\right) = 30 \times 2278 = 68\,340$.

Finally, the sum of the 2017 integers which Ashley has not underlined is $611\,048 - 68\,340 = 542\,708$.

*Solution 2*

We begin by considering the integers from 1 to 60.

When Ashley underlines the integers divisible by 2 and by 5, this will eliminate all of the integers ending in 0, 2, 4, 5, 6, and 8.

This leaves $1, 3, 7, 9, 11, 13, 17, 19, 21, 23, 27, 29, 31, 33, 37, 39, 41, 43, 47, 49, 51, 53, 57, 59$.

Of these, the integers $3, 9, 21, 27, 33, 39, 51, 57$ are divisible by 3.

Therefore, of the first 60 integers, only the integers

$$1, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 49, 53, 59$$

will not be underlined.

Among these 16 integers, we notice that the second set of 8 integers consists of the first 8 integers with 30 added to each.

This pattern continues, so that a corresponding set of 8 out of each block of 30 integers will not be underlined.

Noting that 2010 is the largest multiple of 30 less than 2017, this means that Ashley needs to add the integers

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1 | 7 | 11 | 13 | 17 | 19 | 23 | 29 |
| 31 | 37 | 41 | 43 | 47 | 49 | 53 | 59 |
| 61 | 67 | 71 | 73 | 77 | 79 | 83 | 89 |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| 1981 | 1987 | 1991 | 1993 | 1997 | 1999 | 2003 | 2009 |
| 2011 | 2017 | | | | | | |

Let $S$ equal the sum of these integers.

Before proceeding, we justify briefly why the pattern continues:

> Every positive integer is a multiple of 30, or 1 more than a multiple of 30, or 2 more than a multiple of 30, and so on, up to 29 more than a multiple of 30. Algebraically, this is saying that every positive integer can be written in one of the forms
>
> $$30k, 30k + 1, 30k + 2, 30k + 3, \ldots, 30k + 27, 30k + 28, 30k + 29$$
>
> depending on its remainder when divided by 30.
>
> Every integer with an even remainder when divided by 30 is even, since 30 is also even.
>
> Similarly, every integer with a remainder divisible by 3 or 5 when divided by 30 is divisible by 3 or 5, respectively.
>
> This leaves us with the forms
>
> $$30k + 1, 30k + 7, 30k + 11, 30k + 13, 30k + 17, 30k + 19, 30k + 23, 30k + 29.$$
>
> No integer having one of these forms will be underlined, since, for example, $30k + 11$ is one more than a multiple of 2 and 5 (namely, $30k + 10$) and is 2 more than a multiple of 3 (namely, $30k + 9$) so is not divisible by 2, 3 or 5.

The sum of the 8 integers in the first row of the table above is 120.
Since each of the integers in the second row of the table is 30 greater than the corresponding integer in the first row, then the sum of the numbers in the second row of the table is $120+8\times 30$.
Similarly, the sum of the integers in the third row is $120 + 8 \times 60$, and so on.
We note that $2010 = 67 \times 30$ and $1980 = 66 \times 30$, so there are 67 complete rows in the table.
Therefore,

$$\begin{aligned} S &= 120 + (120 + 8 \times 30) + (120 + 8 \times 60) + \cdots + (120 + 8 \times 1980) + (2011 + 2017) \\ &= 120 \times 67 + 8 \times (30 + 60 + \cdots + 1980) + 4028 \\ &= 8040 + 8 \times 30 \times (1 + 2 + \cdots + 65 + 66) + 4028 \\ &= 12\,068 + 240 \times (33 \times 67) \\ &= 12\,068 + 530\,640 \\ &= 542\,708 \end{aligned}$$

Here, we have used the fact that the integers from 1 to 66 can be grouped into 33 pairs each of which adds to 67, as shown here:

$$1 + 2 + \cdots + 65 + 66 = (1 + 66) + (2 + 65) + \cdots + (33 + 34) = 67 + 67 + \cdots + 67 = 33 \times 67$$

Answer: (A)