The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
cemc.uwaterloo.ca

Gauss Contest

Grade 7

(The Grade 8 Contest is on the reverse side)

Wednesday, May 16, 2018
(in North America and South America)

Thursday, May 17, 2018
(outside of North America and South America)

UNIVERSITY OF
WATERLOO

Time: 1 hour

Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) previously stored information such as formulas, programs, notes, etc., (iv) a computer algebra system, (v) dynamic geometry software.

Instructions

1. Do not open the contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your answer sheet. If you are not sure, ask your teacher to explain it.
4. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. When you have made your choice, enter the appropriate letter for that question on your answer sheet.
5. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
6. Diagrams are *not* drawn to scale. They are intended as aids only.
7. When your supervisor instructs you to start, you will have *sixty* minutes of working time.

The name, school and location of some top-scoring students will be published on the Web site, cemc.uwaterloo.ca. You will also be able to find copies of past Contests and excellent resources for enrichment, problem solving and contest preparation.

Grade 7

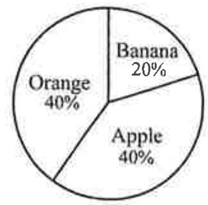
Scoring:	There is *no penalty* for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. What number should be subtracted from 21 to give 8?

   (A) 12 (B) 13 (C) 14 (D) 15 (E) 16

2. In the diagram, the pie chart shows the results of a survey asking students to choose their favourite fruit. 100 students were surveyed. How many students chose banana?



   (A) 40 (B) 80 (C) 100
   (D) 20 (E) 60

3. A class begins at 8:30 a.m. and ends at 9:05 a.m. on the same day. In minutes, what is the length of the class?

   (A) 15 (B) 25 (C) 35 (D) 45 (E) 75

4. A square has an area of 144 cm^2. The side length of the square is

   (A) 288 cm (B) 72 cm (C) 48 cm (D) 12 cm (E) 36 cm

5. If there is no tax, which of the following costs more than \$18 to purchase?

   (A) Five \$1 items and five \$2 items
   (B) Nine \$1 items and four \$2 items
   (C) Nine \$1 items and five \$2 items
   (D) Two \$1 items and six \$2 items
   (E) Sixteen \$1 items and no \$2 items

6. Which of the following numbers lies between 3 and 4 on a number line?

   (A) $\frac{5}{2}$ (B) $\frac{11}{4}$ (C) $\frac{11}{5}$ (D) $\frac{13}{4}$ (E) $\frac{13}{5}$

7. An envelope contains 2 sunflower seeds, 3 green bean seeds, and 4 pumpkin seeds. Carrie randomly chooses one of the seeds from the envelope. What is the probability that Carrie chooses a sunflower seed?

   (A) $\frac{2}{9}$ (B) $\frac{5}{9}$ (C) $\frac{9}{7}$ (D) $\frac{7}{9}$ (E) $\frac{1}{9}$

8. If $x = 4$ and $y = 3x$, the value of y is

   (A) 12 (B) 24 (C) 7 (D) 81 (E) 4

9. The measure of one angle of an isosceles triangle is 50°. The measures of the other angles in this triangle could be

   (A) 50° and 90° (B) 40° and 50° (C) 50° and 80°
   (D) 30° and 100° (E) 60° and 70°

10. The 26 letters of the alphabet are written in order, clockwise around a circle. The *ciphertext* of a message is created by replacing each letter of the message by the letter that is 4 letters clockwise from the original letter. (This is called a *Caesar cipher.*) For example, the message ZAP has ciphertext DET. What is the ciphertext of the message WIN?

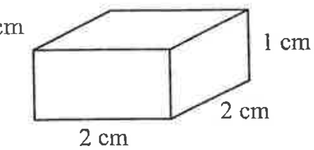
    (A) ALN (B) ZLN (C) AMR (D) AMQ (E) ZMQ

Part B: Each correct answer is worth 6.

11. A cube has exactly six faces and twelve edges. How many vertices does a cube have?

(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

12. What is the surface area of a 1 cm by 2 cm by 2 cm rectangular prism?



(A) 10 cm^2 (B) 20 cm^2 (C) 12 cm^2
(D) 24 cm^2 (E) 16 cm^2

13. At a factory, 11 410 kg of rice is distributed equally into 3260 bags. A family uses 0.25 kg of rice each day. How many days would it take this family to use up one bag of rice?

(A) 9 (B) 12 (C) 13 (D) 14 (E) 15

14. Dalia's birthday is on a Wednesday and Bruce's birthday is 60 days after Dalia's. On what day of the week is Bruce's birthday?

(A) Monday (B) Tuesday (C) Friday (D) Saturday (E) Sunday

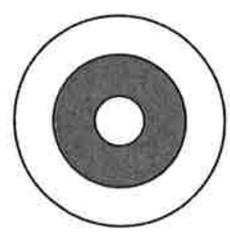
15. Karl has 30 birds. Some of his birds are emus and the rest are chickens. Karl hands out 100 treats to his birds. Each emu gets 2 treats and each chicken gets 4 treats. How many chickens does Karl have?

(A) 10 (B) 15 (C) 25 (D) 20 (E) 6

16. The integers 1 to 32 are spaced evenly and in order around the outside of a circle. Straight lines that pass through the centre of the circle join these numbers in pairs. Which number is paired with 12?

(A) 28 (B) 27 (C) 23 (D) 21 (E) 29

17. In the diagram, the area of the shaded middle ring is 6 times the area of the smallest circle. The area of the unshaded outer ring is 12 times the area of the smallest circle. What fraction of the area of the largest circle is the area of the smallest circle?



(A) $\frac{1}{3}$ (B) $\frac{1}{6}$ (C) $\frac{1}{12}$
(D) $\frac{1}{18}$ (E) $\frac{1}{19}$

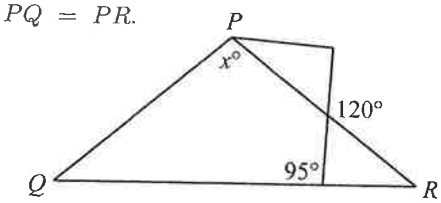
18. There are several groups of six integers whose product is 1. Which of the following cannot be the sum of such a group of six integers?

(A) -6 (B) -2 (C) 0 (D) 2 (E) 6

19. The heights of 4 athletes on a team are 135 cm, 160 cm, 170 cm, and 175 cm. Laurissa joins the team. On the new team of 5 athletes, the mode height of the players is equal to the median height which is equal to the mean (average) height. How tall is Laurissa?

(A) 135 cm (B) 160 cm (C) 170 cm (D) 175 cm (E) 148 cm

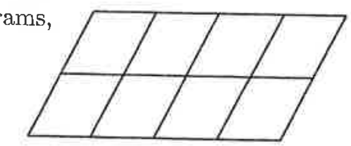
20. In the diagram, $\triangle PQR$ is isosceles with $PQ = PR$. What is the value of x?



(A) 110 **(B)** 90 **(C)** 95
(D) 100 **(E)** 105

Part C: Each correct answer is worth 8.

21. The figure consists of 8 identical small parallelograms, joined as shown. Including these 8 small parallelograms, how many parallelograms appear in this figure?



(A) 29 **(B)** 30 **(C)** 26
(D) 27 **(E)** 28

22. In a jar, there are 50 coins with a total value of \$5.00. The coins are quarters (worth \$0.25 each), dimes (worth \$0.10 each), and nickels (worth \$0.05 each). The number of nickels in the jar is three times the number of quarters. The number of dimes is one more than the number of nickels. How many quarters are in the jar?

(A) 7 **(B)** 6 **(C)** 5 **(D)** 9 **(E)** 8

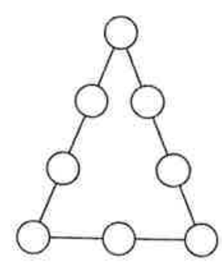
23. The digits from 1 to 9 are written in order so that the digit n is written n times. This forms the block of digits $1223334444\cdots999999999$. The block is written 100 times. What is the 1953^{rd} digit written?

(A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

24. The number 2018 is used to create six-digit positive integers. These six-digit integers must contain the digits 2018 together and in this order. For example, 720 186 is allowed, but 209 318 and 210 893 are not. How many of these six-digit integers are divisible by 9?

(A) 28 **(B)** 27 **(C)** 31 **(D)** 34 **(E)** 22

25. In the triangle, each of the numbers $1, 2, 3, 4, 5, 6, 7, 8$ is placed into a different circle. The sums of the numbers on each of the three sides of the triangle are equal to the same number, S. The sum of all of the different possible values of S is



(A) 85 **(B)** 99 **(C)** 66
(D) 81 **(E)** 67

Grade 7

1. Since $21 - 13 = 8$, the number that should be subtracted from 21 to give 8 is 13.

   ANSWER: (B)

2. Reading from the pie chart, 20% of 100 students chose banana.
   Since 20% of 100 is 20, then 20 students chose banana.

   ANSWER: (D)

3. There are 30 minutes between 8:30 a.m. and 9:00 a.m.
   There are 5 minutes between 9:00 a.m. and 9:05 a.m.
   Therefore, the length of the class is $30 + 5 = 35$ minutes.

   ANSWER: (C)

4. The side length of a square having an area of 144 cm^2 is $\sqrt{144}$ cm or 12 cm.

   ANSWER: (D)

5. The cost of nine \$1 items and five \$2 items is $(9 \times \$1) + (5 \times \$2)$, which is $\$9 + \$10$ or \$19.
   The correct answer is (C).
   (We may check that each of the remaining four answers gives a cost that is less than \$18.)

   ANSWER: (C)

6. Converting each of the improper fractions to a mixed fraction, we get $\frac{5}{2} = 2\frac{1}{2}$, $\frac{11}{4} = 2\frac{3}{4}$, $\frac{11}{5} = 2\frac{1}{5}$, $\frac{13}{4} = 3\frac{1}{4}$, and $\frac{13}{5} = 2\frac{3}{5}$.
   Of the five answers given, the number that lies between 3 and 4 on a number line is $3\frac{1}{4}$ or $\frac{13}{4}$.

   ANSWER: (D)

7. Exactly 2 of the $2 + 3 + 4 = 9$ seeds are sunflower seeds.
   Therefore, the probability that Carrie chooses a sunflower seed is $\frac{2}{9}$.

   ANSWER: (A)

8. Since $x = 4$, then $y = 3 \times 4 = 12$.

   ANSWER: (A)

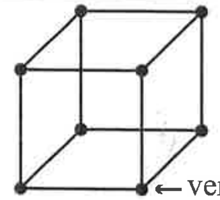
9. The sum of the three angles in any triangle is 180°.
   If one of the angles in an isosceles triangle measures 50°, then the sum of the measures of the two unknown angles in the triangle is $180^\circ - 50^\circ = 130^\circ$.
   Since the triangle is isosceles, then two of the angles in the triangle have equal measure.
   If the two unknown angles are equal in measure, then they each measure $130^\circ \div 2 = 65^\circ$.
   However, 65° and 65° is not one of the given answers.
   If the measure of one of the unknown angles is equal to the measure of the given angle, 50°, then the third angle in the triangle measures $180^\circ - 50^\circ - 50^\circ = 80^\circ$.
   Therefore, the measures of the other angles in this triangle could be 50° and 80°.

   ANSWER: (C)

10. Moving 3 letters clockwise from W, we arrive at the letter Z.
    Moving 1 letter clockwise from the letter Z, the alphabet begins again at A.
    Therefore, the letter that is 4 letters clockwise from W is A.
    Moving 4 letters clockwise from I, we arrive at the letter M.
    Moving 4 letters clockwise from N, we arrive at the letter R.
    The ciphertext of the message WIN is AMR.

    ANSWER: (C)

11. Every cube has exactly 8 vertices, as shown in the diagram.



ANSWER: (E)

12. The area of the 2 cm by 2 cm base of the rectangular prism is $2 \times 2 = 4$ cm^2.
The top face of the prism is identical to the base and so its area is also 4 cm^2.
Each of the 4 vertical faces of the prism has dimensions 2 cm by 1 cm, and thus has area $2 \times 1 = 2$ cm^2.
Therefore the surface area of the rectangular prism is $2 \times 4 + 4 \times 2 = 16$ cm^2.

ANSWER: (E)

13. Since 11 410 kg of rice is distributed into 3260 bags, then each bag contains $11\,410 \div 3260 = 3.5$ kg of rice.
Since a family uses 0.25 kg of rice each day, then it would take this family $3.5 \div 0.25 = 14$ days to use up one bag of rice.

ANSWER: (D)

14. Since Dalia's birthday is on a Wednesday, then any exact number of weeks after Dalia's birthday will also be a Wednesday.
Therefore, exactly 8 weeks after Dalia's birthday is also a Wednesday.
Since there are 7 days in each week, then $7 \times 8 = 56$ days after Dalia's birthday is a Wednesday.
Since 56 days after Dalia's birthday is a Wednesday, then 60 days after Dalia's birthday is a Sunday (since 4 days after Wednesday is Sunday).
Therefore, Bruce's birthday is on a Sunday.

ANSWER: (E)

15. *Solution 1*
Since each emu gets 2 treats and each chicken gets 4 treats, each of Karl's 30 birds gets *at least* 2 treats .
If Karl begins by giving his 30 birds exactly 2 treats each, then Karl will have given out $30 \times 2 = 60$ of the treats.
Since Karl has 100 treats to hand out, then he has $100 - 60 = 40$ treats left to give.
However, each emu has already received their 2 treats (since all 30 birds were given 2 treats).
So the remaining 40 treats must be given to chickens.
Each chicken is to receive 4 treats and has already received 2 treats.
Therefore, each chicken must receive 2 more treats.
Since there are 40 treats remaining, and each chicken receives 2 of these treats, then there are $40 \div 2 = 20$ chickens.
(We may check that if there are 20 chickens, then there are $30 - 20 = 10$ emus, and Karl would then give out $4 \times 20 + 2 \times 10 = 100$ treats.)

Solution 2
Using a trial and error approach, if Karl had 5 emus and $30 - 5 = 25$ chickens, then he would need to hand out $5 \times 2 + 25 \times 4 = 110$ treats.
Since Karl hands out 100 treats, we know that Karl has more emus than 5 (and fewer chickens than 25).
We show this attempt and continue with this approach in the table below.

Number of Emus	Number of Chickens	Number of Emu Treats	Number of Chicken Treats	Total Number of Treats
5	$30-5=25$	$5\times 2=10$	$25\times 4=100$	$10+100=110$
7	$30-7=23$	$7\times 2=14$	$23\times 4=92$	$14+92=106$
10	$30-10=20$	$10\times 2=20$	$20\times 4=80$	$20+80=100$

Therefore, Karl has 20 chickens.

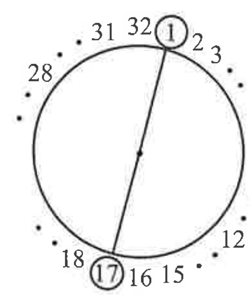
ANSWER: (D)

16. *Solution 1*

The integers 1 to 32 are spaced evenly and in order around the outside of a circle.
Consider drawing a first straight line that passes through the centre of the circle and joins any one pair of these 32 numbers.
This leaves $32-2=30$ numbers still to be paired.
Since this first line passes through the centre of the circle, it divides the circle in half.
In terms of the remaining 30 unpaired numbers, this means that 15 of these numbers lie on each side of the line drawn between the first pair.
Let the number that is paired with 12 be n.
If we draw the line through the centre joining 12 and n, then there are 15 numbers that lie between 12 and n (moving in either direction, clockwise or counter-clockwise).
Beginning at 12 and moving in the direction of the 13, the 15 numbers that lie between 12 and n are the numbers $13, 14, 15, \ldots, 26, 27$.
Therefore, the next number after 27 is the number n that is paired with 12.
The number paired with 12 is 28.

Solution 2

We begin by placing the integers 1 to 32, spaced evenly and in order, clockwise around the outside of a circle.
As in Solution 1, we recognize that there are 15 numbers on each side of the line which joins 1 with its partner.
Moving in a clockwise direction from 1, these 15 numbers are $2, 3, 4, \ldots, 15, 16$, and so 1 is paired with 17, as shown.
Since 2 is one number clockwise from 1, then the partner for 2 must be one number clockwise from 17, which is 18.
Similarly, 12 is 11 numbers clockwise from 1, so the partner for 12 must be 11 numbers clockwise from 17.
Therefore, the number paired with 12 is $17+11=28$.



ANSWER: (A)

17. We may begin by assuming that the area of the smallest circle is 1.
The area of the shaded middle ring is 6 times the area of the smallest circle, and thus has area 6.
The area of the unshaded outer ring is 12 times the area of the smallest circle, and thus has area 12.
The area of the largest circle is the sum of the areas of the smallest circle, the shaded middle ring, and the unshaded outer ring, or $1+6+12=19$.
Therefore, the area of the smallest circle is $\frac{1}{19}$ of the area of the largest circle.
Note: We assumed the area of the smallest circle was 1, however we could have assumed it to have any area. For example, assume the area of the smallest circle is 5 and redo the question. What is your final answer?

ANSWER: (E)

18. For the product of two integers to equal 1, the two integers must both equal 1 or must both equal -1.
Similarly, if the product of six integers is equal to 1, then each of the six integers must equal 1 or -1.
For the product of six integers, each of which is equal to 1 or -1, to equal 1, the number of -1s must be even, because an odd number of -1s would give a product that is negative.
That is, there must be zero, two, four or six -1s among the six integers.
We summarize these four possibilities in the table below.

Number of -1s	Product of the six integers	Sum of the six integers
0	$(1)(1)(1)(1)(1)(1) = 1$	$1+1+1+1+1+1=6$
2	$(-1)(-1)(1)(1)(1)(1) = 1$	$(-1)+(-1)+1+1+1+1=2$
4	$(-1)(-1)(-1)(-1)(1)(1) = 1$	$(-1)+(-1)+(-1)+(-1)+1+1=-2$
6	$(-1)(-1)(-1)(-1)(-1)(-1) = 1$	$(-1)+(-1)+(-1)+(-1)+(-1)+(-1)=-6$

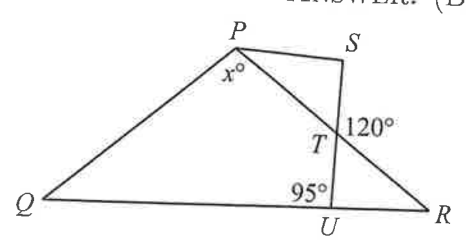
Of the answers given, the sum of such a group of six integers cannot equal 0.

Answer: (C)

19. Since the heights of the 4 athletes on the team are all different, then if Laurissa's height is different than each of these, there is no single mode height.
Therefore, Laurissa's height must be equal to the height of one of the 4 athletes on the team for there to be a single mode.
If Laurissa's height is 135 cm, then the median height of the 5 athletes is 160 cm which is not possible, since the median does not equal the mode.
Similarly, if Laurissa's height is 175 cm, then the median height of the 5 athletes is 170 cm which is not possible.
Therefore, Laurissa's height must equal 160 cm or 170 cm, since in either case the median height of the 5 athletes will equal Laurissa's height, which is the mode.
If Laurissa's height is 170 cm, then the mean height of the 5 athletes is
$\dfrac{135+160+170+170+175}{5} = 162$ cm.
If Laurissa's height is 160 cm, then the mean height of the 5 athletes is
$\dfrac{135+160+160+170+175}{5} = 160$ cm.
When Laurissa's height is 160 cm, the heights of the 5 athletes (measured in cm) are: 135, 160, 160, 170, 175.
In this case, each of the mode, median and mean height of the 5 athletes equals 160 cm.

Answer: (B)

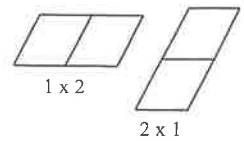
20. We begin by labelling points S, T and U, as shown.



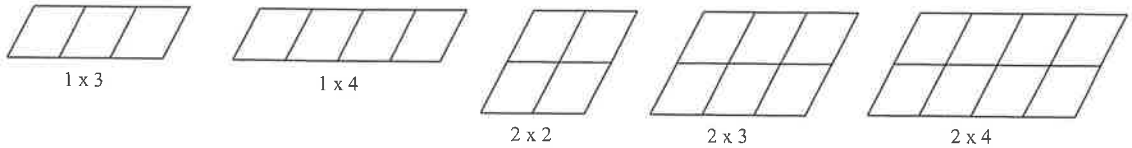
Since S, T, U lie on a straight line, $\angle STU$ measures 180°.
Therefore, $\angle RTU = 180^\circ - \angle STR = 180^\circ - 120^\circ = 60^\circ$.
Similarly, Q, U, R lie on a straight line, and so $\angle QUR$ measures 180°.
Therefore, $\angle TUR = 180^\circ - \angle TUQ = 180^\circ - 95^\circ = 85^\circ$.
The sum of the angles in ΔTUR is 180°.
Thus, $\angle TRU = 180^\circ - \angle RTU - \angle TUR = 180^\circ - 60^\circ - 85^\circ = 35^\circ$.
Since ΔPQR is isosceles with $PQ = PR$, then $\angle PQR = \angle PRQ = 35^\circ$.
Finally, the sum of the angles in ΔPQR is 180°, and so $x^\circ = 180^\circ - \angle PQR - \angle PRQ$ or $x^\circ = 180^\circ - 35^\circ - 35^\circ$ and so $x = 110$.

Answer: (A)

21. The figure formed by combining a pair of adjacent small parallelograms, is also a parallelogram.
For example, each of the two figures shown is a parallelogram.
The reason for this is that opposite sides of these new figures are equal in length and they are parallel. We use the notation $a \times b$ to mean that the new figure has a rows of the small parallelograms and b columns of the small parallelograms.



Similarly, more than 2 small parallelograms can be combined to form new parallelograms.
In addition to the small parallelogram (1×1) and the 1×2 and 2×1 parallelograms shown above, the sizes of the remaining parallelograms that appear in the figure are shown below.



In the table below, the number of parallelograms of each of the different sizes is shown.

Size	1×1	1×2	2×1	1×3	1×4	2×2	2×3	2×4
Number of Parallelograms	8	6	4	4	2	3	2	1

The number of parallelograms appearing in the figure is $8 + 6 + 4 + 4 + 2 + 3 + 2 + 1 = 30$.

ANSWER: (B)

22. *Solution 1*

The number of dimes in the jar is one more than the number of nickels.
If we remove one dime from the jar, then the number of coins remaining in the jar is $50 - 1 = 49$, and the value of the coins remaining in the jar is \$5.00 – \$0.10 = \$4.90.
Also, the number of nickels remaining in the jar is now equal to the number of nickels remaining in the jar, and the number of nickels remaining in the jar is three times the number of quarters remaining in the jar.
That is, for every 1 quarter remaining in the jar, there are 3 nickels and 3 dimes.
Consider groups consisting of exactly 1 quarter, 3 nickels and 3 dimes.
In each of these groups, there are 7 coins whose total value is $\$0.25 + 3 \times \$0.05 + 3 \times \$0.10 = \$0.25 + \$0.15 + \$0.30 = \$0.70$.
Since there are 49 coins having a value of \$4.90 remaining in the jar, then there must be 7 such groups of 7 coins remaining in the jar (since $7 \times 7 = 49$).
(We may check that 7 such groups of coins, with each group having a value of \$0.70, has a total value of $7 \times \$0.70 = \$4.90$, as required.)
Therefore, there are 7 quarters in the jar.

Solution 2

To find the number of quarters in the jar, we need only focus on the total number of coins in the jar, 50, or on the total value of the coins in the jar, \$5.00.
In the solution that follows, we consider both the number of coins in the jar as well as the value of the coins in the jar, to demonstrate that each approach leads to the same answer.
We use a trial and error approach.
Suppose that the number of quarters in the jar is 5 (the smallest of the possible answers given).
The value of 5 quarters is $5 \times 25¢ = 125¢$.
Since the number of nickels in the jar is three times the number of quarters, there would be $3 \times 5 = 15$ nickels in the jar.
The value of 15 nickels is $15 \times 5¢ = 75¢$.

Since the number of dimes in the jar is one more than the number of nickels, there would be $15 + 1 = 16$ dimes in the jar.
The value of 16 dimes is $16 \times 10¢ = 160¢$.
If there were 5 quarters in the jar, then the total number of coins in the jar would be $5 + 15 + 16 = 36$, and so there must be more than 5 quarters in the jar.
Similarly, if there were 5 quarters in the jar, then the total value of the coins in the jar would be $125¢ + 75¢ + 160¢ = 360¢$.
Since the value of the coins in the jar is \$5.00 or 500¢ then the number of quarters in the jar is greater than 5.
We summarize our next two trials in the table below.

Number of Quarters	Value of Quarters	Number of Nickels	Value of Nickels	Number of Dimes	Value of Dimes	Total Value of Coins
6	150¢	18	90¢	19	190¢	430¢
7	175¢	21	105¢	22	220¢	500¢

When there are 7 quarters in the jar, there are $7 + 21 + 22 = 50$ coins in the jar, as required.
When there are 7 quarters in the jar, the value of the coins in the jar is $175¢ + 105¢ + 220¢ = 500¢$ or \$5.00, as required.
In either case, the number of quarters in the jar is 7.

ANSWER: (A)

23. In each block $1223334444 \cdots 999999999$, there is 1 digit 1, 2 digits 2, 3 digits 3, and so on.
The total number of digits written in each block is $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = 45$.
We note that $1953 \div 45$ gives a quotient of 43 and a remainder of 18 (that is, $1953 = 45 \times 43 + 18$).
Since each block contains 45 digits, then 43 blocks contain $43 \times 45 = 1935$ digits.
Since $1953 - 1935 = 18$, then the 18^{th} digit written in the next block (the 44^{th} block) will be the 1953^{rd} digit written.
Writing out the first 18 digits in a block, we get 122333444455555666, and so the 1953^{rd} digit written is a 6.

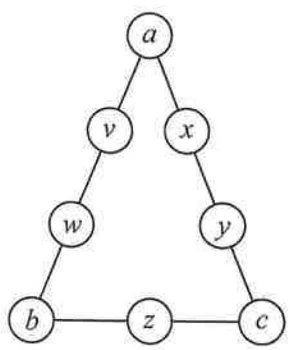
ANSWER: (C)

24. For a positive integer to be divisible by 9, the sum of its digits must be divisible by 9.
In this problem, we want to count the number of six-digit positive integers containing 2018 and divisible by 9.
Thus, we must find the remaining two digits, which together with 2018, form a six-digit positive integer that is divisible by 9.
The digits 2018 have a sum of $2 + 0 + 1 + 8 = 11$.
Let the remaining two digits be a and b so that the six-digit positive integer is $ab2018$ or $ba2018$ or $a2018b$ or $b2018a$ or $2018ab$ or $2018ba$.
When the digits a and b are added to 11, the sum must be divisible by 9.
That is, the sum $a + b + 11$ must be divisible by 9.
The smallest that each of a and b can be is 0, and so the smallest that the sum $a + b + 11$ can be is $0 + 0 + 11 = 11$.
The largest that each of a and b can be is 9, and so the largest that the sum $a + b + 11$ can be is $9 + 9 + 11 = 29$.
The only integers between 11 and 29 that are divisible by 9 are 18 and 27.
Therefore, either $a + b = 18 - 11 = 7$ or $a + b = 27 - 11 = 16$.
If $a + b = 7$, then the digits a and b are 0 and 7 or 1 and 6 or 2 and 5 or 3 and 4, in some order.
If a and b are 1 and 6, then the possible six-digit integers are 162 018, 612 018, 120 186, 620 181,

201 816, and 201 861.
In this case, there are 6 possible six-digit positive integers.
Similarly, if a and b are 2 and 5, then there are 6 possible six-digit integers.
Likewise, if a and b are 3 and 4, then there are 6 possible six-digit integers.
If a and b are 0 and 7, then the possible six-digit integers are 702 018, 720 180, 201 870, and 201 807, since the integer cannot begin with the digit 0.
In this case, there are 4 possible six-digit positive integers.
Therefore, for the case in which the sum of a and b is 7, there are $6 + 6 + 6 + 4 = 22$ possible six-digit integers.
Finally, we consider the case for which the sum of the digits a and b is 16.
If $a + b = 16$, then the digits a and b are 7 and 9, or 8 and 8.
If a and b are 7 and 9, then there are again 6 possible six-digit integers (792 018, 972 018, 720 1869, 920 187, 201 879, and 201 897).
If a and b are 8 and 8, then there are 3 possible six-digit integers: 882 018, 820 188, and 201 888.
Therefore, for the case in which the sum of a and b is 16, there are $6 + 3 = 9$ possible six-digit integers, and so there are $22 + 9 = 31$ six-digit positive integers in total.
We note that all of these 31 six-digit positive integers are different from one another, and that they are the only six-digit positive integers satisfying the given conditions.
Therefore, there are 31 six-digit positive integers that are divisible by 9 and that contain the digits 2018 together and in this order.

ANSWER: (C)

25. We label the unknown numbers in the circles as shown:



Since the sum of the numbers along each side of the triangle is S then

$$S = a + v + w + b \qquad S = a + x + y + c \qquad S = b + z + c$$

When we add the numbers along each of the three sides of the triangles, we include each of a, b and c twice and obtain

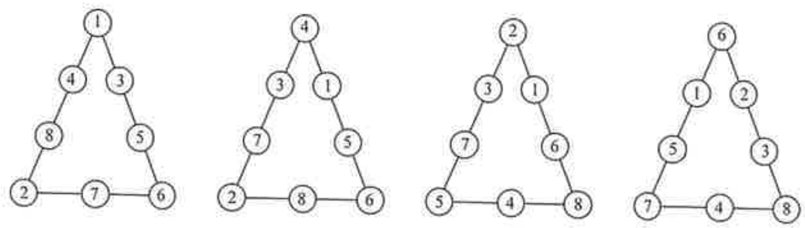
$$S+S+S = (a+v+w+b)+(a+x+y+c)+(b+z+c) = (a+v+w+b+z+c+y+x)+a+b+c$$

Now the numbers a, v, w, b, z, c, y, x are the numbers $1, 2, 3, 4, 5, 6, 7, 8$ in some order.
This means that $a + v + w + b + z + c + y + x = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 = 36$.
Therefore,

$$3S = 36 + a + b + c$$

Since $3S$ is a multiple of 3 and 36 is a multiple of 3, then $a + b + c$ (which equals $3S - 36$) must also be a multiple of 3.
Looking at the possible numbers that can go in the circles, the smallest that $a + b + c$ can be is

$1+2+3$ or 6, which would make $3S = 36 + 6 = 42$ or $S = 14$. S cannot be any smaller than 14 because $a+b+c$ cannnot be any smaller than 6 and so $3S$ cannot be any smaller than 42. Looking at the possible numbers that can go in the circles, the largest that $a+b+c$ can be is $6+7+8$ or 21, which would make $3S = 36 + 21 = 57$ or $S = 19$. S cannot be any larger than 19 because $a+b+c$ cannot be any larger than 21 and so $3S$ cannot be any larger than 57.
So which of the values $S = 14, 15, 16, 17, 18, 19$ is actually possible?
The following diagrams show ways of completing the triangle with $S = 15, 16, 17, 19$:



Coming up with these examples requires a combination of reasoning and fiddling.
For example, consider the case when $S = 15$.
Since $3S = 36 + a + b + c$ and $S = 15$, then $a + b + c = 3 \times 15 - 36 = 9$.
In the given example, we have $a = 1$, $b = 2$ and $c = 6$.
Since the bottom row $(b + z + c)$ has the smallest number of circles, we put the largest of a, b, c here ($b = 2$ and $c = 6$) and then set $z = 15 - b - c = 7$. A bit of fiddling allows us to choose u, v, x, y appropriately to get the desired sums on the two other sides.
We note that there are other possible combinations of a, b, c with $a + b + c = 9$ (namely, $1, 3, 5$ and $2, 3, 4$). It turns out that neither of these possibilities can produce a triangle with $S = 15$.
In a similar way, we can determine examples like those shown with $S = 16, 17, 19$.

To complete the solution, we show that $S = 14$ and $S = 18$ are not possible.
Suppose that $S = 14$.
In this case, $a + b + c = 3S - 36 = 3 \times 14 - 36 = 6$.
The only integers from the list $1, 2, 3, 4, 5, 6, 7, 8$ which give this sum are $1, 2, 3$.
Consider the bottom row, which should have $b + z + c = 14$.
Since a, b, c are $1, 2, 3$ in some order, then $b + c$ is at most $2 + 3 = 5$.
Since the maximum number in the triangle is 8, then z is at most 8.
This makes $b + z + c$ at most $5 + 8 = 13$, which means that $b + z + c$ cannot equal 14.
This means that we cannot build a triangle with $S = 14$.
Suppose that $S = 18$.
In this case, $a + b + c = 3S - 36 = 3 \times 18 - 36 = 18$.
There are several possible sets of values for a, b, c: $3, 7, 8$ and $4, 6, 8$ and $5, 6, 7$.
Consider the bottom row again, which should have $b + z + c = 18$.
Here we have $a + b + c = 18$ and $b + z + c = 18$.
Since b and c are common to these sums and the total is the same in each case, then $a = z$, which is not allowed.
This means that we cannot build a triangle with $S = 18$.

In summary, the possible values of S are $15, 16, 17, 19$.
The sum of these values is 67.

ANSWER: (E)