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The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
cemc.uwaterloo.ca

Gauss Contest

Grade 8

(The Grade 7 Contest is on the reverse side)

Wednesday, May 10, 2017
(in North America and South America)

Thursday, May 11, 2017
(outside of North America and South America)

UNIVERSITY OF WATERLOO

Time: 1 hour

Calculators are allowed, with the following restriction: you may not use a device that has internet access, that can communicate with other devices, or that contains previously stored information. For example, you may not use a smartphone or a tablet.

Instructions

1. Do not open the contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your answer sheet. If you are not sure, ask your teacher to explain it.
4. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. When you have made your choice, enter the appropriate letter for that question on your answer sheet.
5. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
6. Diagrams are *not* drawn to scale. They are intended as aids only.
7. When your supervisor instructs you to start, you will have *sixty* minutes of working time.

The name, school and location of some top-scoring students will be published on the Web site, cemc.uwaterloo.ca. You will also be able to find copies of past Contests and excellent resources for enrichment, problem solving and contest preparation.

Grade 8

Scoring:	There is *no penalty* for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. Michael has \$280 in \$20 bills. How many \$20 bills does he have?

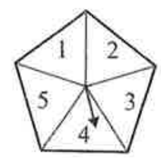
   (A) 10 **(B)** 12 **(C)** 14 **(D)** 16 **(E)** 18

2. The value of $4^2 - 2^3$ is

   (A) 8 **(B)** 2 **(C)** 4 **(D)** 0 **(E)** 6

3. A pentagon is divided into 5 equal sections, as shown. An arrow is attached to the centre of the pentagon. The arrow is spun once. What is the probability that the arrow stops in the section numbered 4?

   (A) $\frac{3}{5}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{4}{5}$
   (D) $\frac{1}{4}$ **(E)** $\frac{1}{5}$



4. There are 160 students in grade 8 at Murray Public School. If exactly 10% of these students are on the school's chess team, how many grade 8 students are on the team?

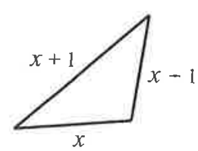
   (A) 26 **(B)** 16 **(C)** 20 **(D)** 12 **(E)** 18

5. 44×22 is equal to

   (A) 88×2 **(B)** 88×11 **(C)** 88×20 **(D)** 88×44 **(E)** 88×40

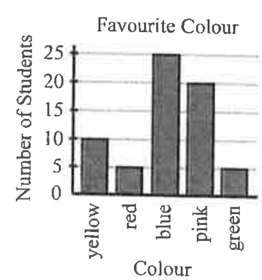
6. If the perimeter of the triangle shown is 21, what is the value of x?

   (A) 3 **(B)** 7 **(C)** 8
   (D) 13 **(E)** 16



7. Students were surveyed about their favourite colour and the results are displayed in the graph shown. What is the ratio of the number of students who chose pink to the number of students who chose blue?

   (A) 4 : 5 **(B)** 3 : 5 **(C)** 1 : 5
   (D) 2 : 5 **(E)** 5 : 3



8. When a number is tripled and then decreased by 6, the result is 15. The number is

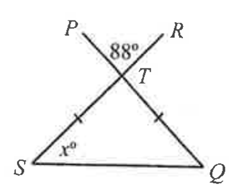
   (A) 8 **(B)** 6 **(C)** 5 **(D)** 7 **(E)** 9

9. Tian measured her steps and found that it took her 625 steps to walk 500 m. If she walks 10 000 steps at this same rate, what distance will she walk?

   (A) 6.4 km **(B)** 6.25 km **(C)** 7.5 km **(D)** 8 km **(E)** 7.2 km

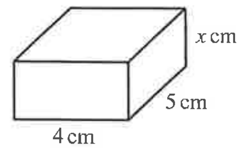
10. Line segments PQ and RS intersect at T, as shown. If $TS = TQ$ and $\angle PTR = 88^\circ$, the value of x is

    (A) 44 **(B)** 46 **(C)** 88
    (D) 45 **(E)** 50



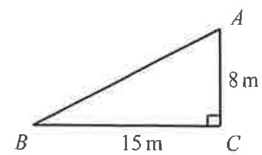
Part B: Each correct answer is worth 6.

11. The volume of the rectangular prism shown is 60 cm^3. What is the value of x?



(A) 1 (B) 4 (C) 6
(D) 3 (E) 2

12. In the diagram shown, David begins at A and walks in a straight line to C, and then walks in a straight line from C to B. Cindy also begins at A and walks in a straight line to B. How much farther does David walk than Cindy?



(A) 0 m (B) 2 m (C) 4 m
(D) 6 m (E) 7 m

13. The sum of the first 100 positive integers (that is, $1+2+3+\cdots+99+100$) equals 5050. The sum of the first 100 positive multiples of 10 (that is, $10+20+30+\cdots+990+1000$) equals

(A) 10 100 (B) 5950 (C) 50 500 (D) 6050 (E) 45 450

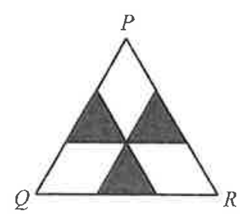
14. There are 20 pens to be given away to 4 students. Each student receives a different number of pens and each student receives at least one pen. What is the largest number of pens that a student can receive?

(A) 17 (B) 15 (C) 14 (D) 8 (E) 5

15. The number of even integers between 1 and 103 is the same as the number of odd integers between 4 and

(A) 104 (B) 102 (C) 100 (D) 108 (E) 106

16. In the diagram, $\triangle PQR$ is equilateral and has side length 6 cm. Each of the shaded triangles is equilateral and has side length 2 cm. What fraction of the area of $\triangle PQR$ is shaded?



(A) $\frac{3}{7}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$
(D) $\frac{3}{5}$ (E) $\frac{2}{3}$

17. On coach Wooden's basketball team:
    - Meghan is the tallest player,
    - Meghan's height is 188 cm, and
    - Avery is the shortest player.

    When used with the information above, which of the following single statements is enough to determine Avery's height?

    (A) The median of the players' heights is 170 cm
    (B) The mode of the players' heights is 160 cm
    (C) The mean of the players' heights is 165 cm
    (D) The range of the players' heights is 33 cm
    (E) There are 10 players on the team

18. Brodie and Ryan are driving directly towards each other. Brodie is driving at a constant speed of 50 km/h. Ryan is driving at a constant speed of 40 km/h. If they are 120 km apart, how long will it take before they meet?

(A) 1 h 12 min **(B)** 1 h 25 min **(C)** 1 h 15 min **(D)** 1 h 33 min **(E)** 1 h 20 min

19. In a group of seven friends, the mean (average) age of three of the friends is 12 years and 3 months and the mean age of the remaining four friends is 13 years and 5 months. In months, the mean age of all seven friends is

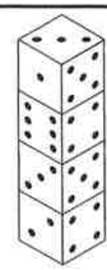
(A) 156 **(B)** 154 **(C)** $155\frac{1}{2}$ **(D)** 157 **(E)** 155

20. In the six-digit number $1ABCDE$, each letter represents a digit. Given that $1ABCDE \times 3 = ABCDE1$, the value of $A + B + C + D + E$ is

(A) 29 **(B)** 26 **(C)** 22 **(D)** 30 **(E)** 28

Part C: Each correct answer is worth 8.

21. The number of dots on opposite faces of a regular die add to 7. Four regular dice are arranged as shown. Which of the following could be the sum of the number of dots hidden between the dice?



(A) 22 **(B)** 26 **(C)** 24
(D) 21 **(E)** 23

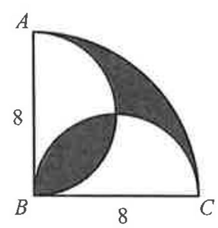
22. The values 2, 3, 4, and 5 are each assigned to exactly one of the letters V, W, X, and Y to give $Y^X - W^V$ the greatest possible value. The value of $X + V$ is equal to

(A) 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

23. Mike and Alain play a game in which each player is equally likely to win. The first player to win three games becomes the champion, and no further games are played. If Mike has won the first game, what is the probability that Mike becomes the champion?

(A) $\frac{1}{4}$ **(B)** $\frac{5}{8}$ **(C)** $\frac{11}{16}$ **(D)** $\frac{3}{5}$ **(E)** $\frac{3}{4}$

24. In the diagram, ABC is a quarter of a circle with radius 8. A semi-circle with diameter AB is drawn, as shown. A second semi-circle with diameter BC is also drawn. The area of the shaded region is closest to



(A) 22.3 **(B)** 33.5 **(C)** 25.1
(D) 18.3 **(E)** 20.3

25. Brady is stacking 600 plates in a single stack. Each plate is coloured black, gold or red. Any black plates are always stacked below any gold plates, which are always stacked below any red plates. The total number of black plates is always a multiple of two, the total number of gold plates is always a multiple of three, and the total number of red plates is always a multiple of six. For example, the plates could be stacked with:

- 180 black plates below 300 gold plates below 120 red plates, or
- 450 black plates below 150 red plates, or
- 600 gold plates.

In how many different ways could Brady stack the plates?

(A) 5139 **(B)** 5142 **(C)** 5145 **(D)** 5148 **(E)** 5151

Grade 8

1. Michael has \$280 in \$20 bills and so the number of \$20 bills that he has is $280 \div 20 = 14$.
   ANSWER: (C)

2. Evaluating, we get $4^2 - 2^3 = 16 - 8 = 8$.
   ANSWER: (A)

3. Exactly 1 of the 5 equal sections contains the number 4.
   Therefore, the probability that the spinner lands on 4 is $\frac{1}{5}$.
   ANSWER: (E)

4. The number of grade 8 students on the chess team is $160 \times 10\% = 160 \times \frac{10}{100} = 160 \times 0.10 = 16$.
   ANSWER: (B)

5. Since $22 = 2 \times 11$, then $44 \times 22 = 44 \times 2 \times 11$.
   Since $44 \times 2 = 88$, then $44 \times 2 \times 11 = 88 \times 11$.
   Therefore, $44 \times 22 = 88 \times 11$.
   ANSWER: (B)

6. In terms of x, the sum of the three side lengths of the triangle is $x + x + 1 + x - 1 = 3x$.
   Since the perimeter is 21, then $3x = 21$ and so $x = 7$.
   ANSWER: (B)

7. Reading from the graph, 20 students chose pink and 25 students chose blue.
   The ratio of the number of students who chose pink to the number of students who chose blue is $20 : 25$.
   After simplifying this ratio (dividing each number by 5), $20 : 25$ is equal to $4 : 5$.
   ANSWER: (A)

8. *Solution 1*
   To get the original number, we reverse the steps.
   That is, we add 6 and then divide by 3.
   Therefore, the original number is $(15 + 6) \div 3 = 21 \div 3 = 7$.

   Solution 2
   If the original number is x, then when it is tripled the result is $3x$.
   When this result is decreased by 6, we get $3x - 6$.
   Solving $3x - 6 = 15$, we get $3x = 15 + 6$ or $3x = 21$ and so $x = 7$.
   ANSWER: (D)

9. Tian travels 500 m every 625 steps and so she travels $500 \div 625 = 0.8$ m with each step.
   If Tian walks 10 000 steps at this same rate, she will walk a distance of $0.8 \times 10\,000 = 8000$ m.
   Since there are 1000 m in each kilometre, Tian will walk $8000 \div 1000 = 8$ km.
   ANSWER: (D)

10. Line segments PQ and RS intersect at T and so $\angle PTR$ and $\angle STQ$ are opposite angles and therefore $\angle STQ = \angle PTR = 88^\circ$.
    Since $TS = TQ$, then $\triangle STQ$ is an isosceles triangle and so $\angle TSQ = \angle TQS = x^\circ$.
    The three interior angles of any triangle add to 180°.
    Thus, $88^\circ + x^\circ + x^\circ = 180^\circ$, and so $2x = 180 - 88$ or $2x = 92$ which gives $x = 46$.
    ANSWER: (B)

11. The volume of a rectangular prism is determined by multiplying the area of its base by its height.
The area of the base for the given prism is $4 \times 5 = 20$ cm^2 and its height is x cm.
Since the prism's volume is 60 cm^3, then $20x = 60$ and so $x = 3$.

ANSWER: (D)

12. Since $\angle ACB = 90°$, then $\triangle ACB$ is a right-angled triangle.
By the Pythagorean Theorem, $AB^2 = AC^2 + CB^2 = 8^2 + 15^2 = 64 + 225 = 289$ m^2.
Since $AB > 0$, then $AB = \sqrt{289} = 17$ m and so Cindy walks a distance of 17 m.
Walking from A to C to B, David walks a total distance of $8 + 15 = 23$ m.
Thus, David walks $23 - 17 = 6$ m farther than Cindy.

ANSWER: (D)

13. Each term of the sum $10 + 20 + 30 + \cdots + 990 + 1000$ is 10 times larger than its corresponding term in the sum $1 + 2 + 3 + \cdots + 99 + 100$, and so the required sum is 10 times larger than the given sum.
Since $1+2+3+\cdots+99+100 = 5050$, then $10+20+30+\cdots+990+1000 = 5050 \times 10 = 50\,500$.

ANSWER: (C)

14. If three of the students receive the smallest total number of pens possible, then the remaining student will receive the largest number of pens possible.
The smallest number of pens that a student can receive is 1, since each student receives at least 1 pen.
Since each student receives a different number of pens, the second smallest number of pens that a student can receive is 2 and the third smallest number of pens that a student can receive is 3.
The smallest total number of pens that three students can receive is $1 + 2 + 3 = 6$.
Therefore, the largest number of pens that a student can receive is $20 - 6 = 14$.

ANSWER: (C)

15. The even integers between 1 and 103 are $2 = 2 \times 1, 4 = 2 \times 2, 6 = 2 \times 3, 8 = 2 \times 4$, and so on up to and including $102 = 2 \times 51$.
Since there are 51 even integers in the list $2, 4, 6, \ldots, 100, 102$, then there are 51 even integers between 1 and 103.
Next, we want to find a number N such that there are 51 odd integers between 4 and N.
We notice that our lower bound, 4, is 3 greater than our original lower bound of 1.
By increasing each of the 51 even integers from above by 3, we create the first 51 odd integers which are greater than 4.
These odd integers are $2 \times 1 + 3 = 5, 2 \times 2 + 3 = 7, 2 \times 3 + 3 = 9, 2 \times 4 + 3 = 11$, and so on up to and including $2 \times 51 + 3 = 105$.
Since there are 51 odd integers in the list $5, 7, 9, \ldots, 103, 105$, then there are 51 odd integers between 4 and 106.
That is, the number of even integers between 1 and 103 is the same as the number of odd integers between 4 and 106.

ANSWER: (E)

16. Label points S, T, U, V, W, as shown in Figure 1.
Each shaded triangle is equilateral, $\triangle PQR$ is equilateral, and so $\angle VSU = \angle VTW = \angle SPT = 60°$.
Therefore, $\angle PSV = 180° - \angle VSU = 120°$ (since PSU is a straight angle), and similarly, $\angle PTV = 120°$.
In quadrilateral $PSVT$, $\angle SVT = 360° - \angle PSV - \angle SPT - \angle PTV$ or $\angle SVT = 360° - 120° - 60° - 120° = 60°$.
Therefore, $PSVT$ is a parallelogram, and since $SV = TV = 2$, then $PS = PT = 2$ (opposite sides of a parallelogram are equal in length).
Join S to T, as shown in Figure 2.
Since $SV = TV$, then $\angle VST = \angle VTS = \frac{1}{2}(180° - 60°) = 60°$.
That is, $\triangle SVT$ is equilateral with side length 2, and is therefore congruent to each of the shaded triangles.
Similarly, $\angle SPT = 60°$, $PS = PT = 2$, and so $\triangle PST$ is congruent to each of the shaded triangles.
We may also join U to X and W to Y (as in Figure 3), and similarly show that $\triangle UQX, \triangle UXV, \triangle WRY$, and $\triangle WYV$ are also congruent to the shaded triangles.
Thus, $\triangle PQR$ can be divided into 9 congruent triangles.
Since 3 of these 9 triangles are shaded, the fraction of the area of $\triangle PQR$ that is shaded is $\frac{3}{9}$ or $\frac{1}{3}$.

Figure 1

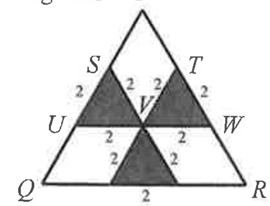


Figure 2

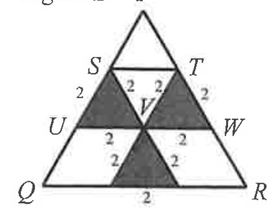
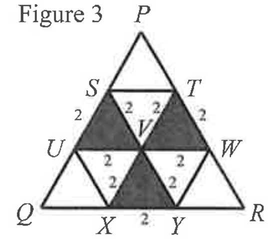


Figure 3



ANSWER: (B)

17. The range of the players' heights is equal to the difference between the height of the tallest player and the height of the shortest player.
Since the tallest player, Meghan, has a height of 188 cm, and the range of the players' heights is 33 cm, then the shortest player, Avery, has a height of $188 - 33 = 155$ cm.
Thus, answer (D) is a statement which provides enough information to determine Avery's height, and so must be the only one of the five statements which is enough to determine Avery's height.
(Can you give a reason why each of the other four answers does not provide enough information to determine Avery's height?)

ANSWER: (D)

18. When Brodie and Ryan are driving directly towards each other at constant speeds of 50 km/h and 40 km/h respectively, then the distance between them is decreasing at a rate of $50 + 40 = 90$ km/h.
If Brodie and Ryan are 120 km apart and the distance between them is decreasing at 90 km/h, then they will meet after $\dfrac{120}{90}$ h or $\dfrac{4}{3}$ h or $1\frac{1}{3}$ h.
Since $\frac{1}{3}$ of an hour is $\frac{1}{3} \times 60 = 20$ minutes, then it will take Brodie and Ryan 1 h 20 min to meet.

ANSWER: (E)

19. The mean age of three of the friends is 12 years and 3 months which is equal to $12 \times 12 + 3 = 144 + 3 = 147$ months.
Since the mean equals the sum of the ages divided by 3, then the sum of the ages of these three friends is $3 \times 147 = 441$ months.
The mean age of the remaining four friends is 13 years and 5 months or $12 \times 13 + 5 = 156 + 5 = 161$ months.

Thus, the sum of the ages of these four friends is $4 \times 161 = 644$ months.
The sum of the ages of all seven friends is $441 + 644 = 1085$ months, and so the mean age of all seven friends is $\frac{1085}{7} = 155$ months.

ANSWER: (E)

20. The units digit of the product $1ABCDE \times 3$ is 1, and so the units digit of $E \times 3$ must equal 1.
Therefore, the only possible value of E is 7.
Substituting $E = 7$, we get

$$\begin{array}{r} 1ABCD7 \\ \times \quad\quad 3 \\ \hline ABCD71 \end{array}$$

Since $7 \times 3 = 21$, 2 is carried to the tens column.
Thus, the units digit of $D \times 3 + 2$ is 7, and so the units digit of $D \times 3$ is 5.
Therefore, the only possible value of D is 5.
Substituting $D = 5$, we get

$$\begin{array}{r} 1ABC57 \\ \times \quad\quad 3 \\ \hline ABC571 \end{array}$$

Since $5 \times 3 = 15$, 1 is carried to the hundreds column.
Thus, the units digit of $C \times 3 + 1$ is 5, and so the units digit of $C \times 3$ is 4.
Therefore, the only possible value of C is 8.
Substituting $C = 8$, we get

$$\begin{array}{r} 1AB857 \\ \times \quad\quad 3 \\ \hline AB8571 \end{array}$$

Since $8 \times 3 = 24$, 2 is carried to the thousands column.
Thus, the units digit of $B \times 3 + 2$ is 8, and so the units digit of $B \times 3$ is 6.
Therefore, the only possible value of B is 2.
Substituting $B = 2$, we get

$$\begin{array}{r} 1A2857 \\ \times \quad\quad 3 \\ \hline A28571 \end{array}$$

Since $2 \times 3 = 6$, there is no carry to the ten thousands column.
Thus, the units digit of $A \times 3$ is 2.
Therefore, the only possible value of A is 4.
Substituting $A = 4$, we get

$$\begin{array}{r} 142857 \\ \times \quad\quad 3 \\ \hline 428571 \end{array}$$

Checking, we see that the product is correct and so $A+B+C+D+E = 4+2+8+5+7 = 26$.

ANSWER: (B)

21. On the bottom die, the two visible faces are showing 2 dots and 4 dots.
Since the number of dots on opposite faces of this die add to 7, then there are 5 dots on the face opposite the face having 2 dots, and 3 dots on the face opposite the face having 4 dots.

Therefore, the top face of this bottom die (which is a face that is hidden between the dice) has either 1 dot on it or it has 6 dots on it.

On the second die from the bottom, the sum of the number of dots on the top and bottom faces (the faces hidden between the dice) is 7 since the number of dots on opposite faces add to 7. (We do not need to know which faces these are, though we could determine that they must have 1 and 6 dots on them.)

Similarly, on the third die from the bottom, the sum of the number of dots on the top and bottom faces (the faces hidden between the dice) is also 7.

Finally, the top face of the top die shows 3 dots, and so the bottom face of this die (which is a face that is hidden between the dice) contains 4 dots.

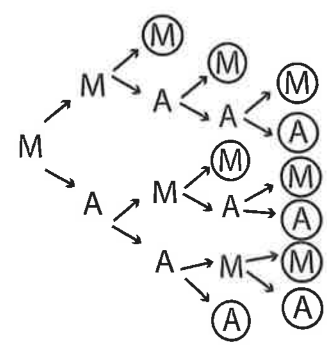
Therefore, the sum of the number of dots hidden between the dice is either $4 + 7 + 7 + 1 = 19$ or $4 + 7 + 7 + 6 = 24$.
Of these two possible answers, 24 is the only answer which appears among the five given answers.

Answer: (C)

22. To give $Y^X - W^V$ the greatest possible value, we make Y^X as large as possible while making W^V as small as possible.
To make Y^X as large as possible, we make Y and X as large as possible.
Thus, we must assign Y and X the two largest values, and assign W and V the two smallest values.
Let Y and X equal 4 and 5 in some order.
Since $4^5 = 1024$ and $5^4 = 625$, we let $Y = 4$ and $X = 5$ so that Y^X is as large as possible.
Similarly, we assign W and V the smallest possible numbers, 2 and 3.
Since $2^3 = 8$ and $3^2 = 9$, we let $W = 2$ and $V = 3$ so that W^V is as small as possible.
Thus, the greatest possible value of $Y^X - W^V$ is equal to $4^5 - 2^3 = 1024 - 8 = 1016$, which gives $X + V = 5 + 3 = 8$.

Answer: (D)

23. We introduce the letter M to represent a game that Mike has won, and the letter A to represent a game that Alain has won.
We construct a tree diagram to show all possible outcomes.
The M at the far left of the tree represents the fact that Mike won the first game.
The second "column" shows the two possible outcomes for the second game - Mike could win (M), or Alain could win (A).
The third, fourth and fifth columns show the possible outcomes of games 3, 4 and 5, respectively.
A branch of the tree is linked by arrows and each branch gives the outcomes of the games which lead to one of the players becoming the champion.



Once Mike has won 3 games, or Alain has won 3 games, the branch ends and the outcome of that final game is circled.
Since the first player to win 3 games becomes the champion, one way that Mike could become the champion is to win the second and third games (since he has already won the first game).
We represent this possibility as MMM, as shown along the top branch of the tree diagram.
In this MMM possibility, the final M is circled in the diagram, meaning that Mike has become the champion.
Since we are asked to determine the probability that Mike becomes the champion, we search

for all branches through the tree diagram which contain 3 Ms (paths ending with a circled M).
The tree diagram shows six such branches: MMM, MMAM, MMAAM, MAMM, MAMAM, and MAAMM.

All other branches end with a circled A, meaning that Alain has won 3 games and becomes the champion.

Since each player is equally likely to win a game, then Mike wins a game with probability $\frac{1}{2}$, and Alain wins a game with probability $\frac{1}{2}$.

Of the six ways that Mike can win (listed above), only one of these ends after 3 games (MMM).
The probability that Mike wins in exactly 3 games is equal to the probability that Mike wins the second game, which is $\frac{1}{2}$, multiplied by the probability that Mike wins the third game, which is also $\frac{1}{2}$.

That is, the probability that Mike becomes the champion by winning the first 3 games is $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$.

Of the six ways that Mike can win, two of these end after 4 games (MMAM, MAMM).
The probability that Mike wins games two and four but Alain wins game three (MMAM) is equal to the probability that Mike wins the second game, which is $\frac{1}{2}$, multiplied by the probability that Alain wins the third game, which is also $\frac{1}{2}$, multiplied by probability that Mike wins the fourth game, $\frac{1}{2}$.

In this case, Mike becomes the champion with probability $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$.

Similary, the probability that Mike becomes the champion by winning games three and four, but loses game two, is also $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$.

Finally, we determine the probabilty that Mike becomes the champion by winning in exactly 5 games (there are three possibilities: MMAAM, MAMAM and MAAMM).

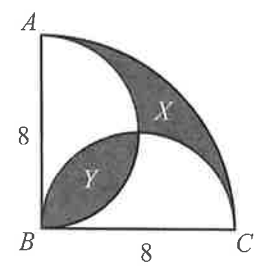
Each of these three possibilities happens with probability $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{16}$.

Therefore, if Mike has won the first game, then the probability that he becomes champion is $\frac{1}{4} + 2 \times \frac{1}{8} + 3 \times \frac{1}{16} = \frac{1}{4} + \frac{2}{8} + \frac{3}{16} = \frac{4+4+3}{16} = \frac{11}{16}$.

ANSWER: (C)

24. Let the area of the shaded region that lies outside of both semi-circles be X.

Let the area of the shaded region that lies inside of both semi-circles be Y.



The sum of the areas of both semi-circles counts the shaded area Y twice (since the area of overlap of the semi-circles is Y).

Therefore, if we subtract Y from the sum of the areas of both semi-circles, and add X, we get the area of the quarter-circle ABC.

That is, the area of quarter-circle ABC is equal to
(the area of the semi-circle drawn on AB)+(the area of the semi-circle drawn on BC)$-Y+X$.

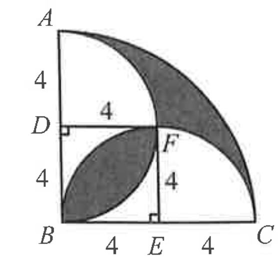
The area of quarter-circle ABC is $\frac{1}{4}\pi(8)^2 = 16\pi$.

The area of the semi-circle drawn on AB is $\frac{1}{2}\pi(4)^2 = 8\pi$.

The area of the semi-circle drawn on BC is also 8π.

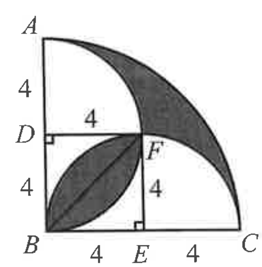
Thus, $16\pi = 8\pi + 8\pi - Y + X$ or $16\pi = 16\pi - Y + X$, and so $Y = X$.

We build square $DBEF$ so that D is 4 vertical units from B, and E is 4 horizontal units to the right of B. Next, we will show that F lies on both semi-circles.



Since ABC is a quarter of a circle, then $\angle ABC = 90°$.
Beginning at point B, we move up vertically 4 units to point D, and then move right 4 units in a direction perpendicular to AB.
After these two moves, we arrive at the point labelled F.
Since the diameter of the semi-circle drawn on AB has length 8, then the radius of this semi-circle is 4.
Therefore, D is the centre of this semi-circle (since $DB = 4$), and F lies on this semi-circle (since $DF = 4$).
Beginning again at point B, we move right 4 units to point E, and then move up vertically 4 units in a direction perpendicular to BC.
Since these are the same two moves we made previously (up 4 and right 4), then we must again arrive at F.
Since the diameter of the semi-circle drawn on BC has length 8, then the radius of this semi-circle is 4.
Therefore, E is the centre of this semi-circle (since $EB = 4$), and F lies on this semi-circle (since $EF = 4$).
The two semi-circles intersect at exactly one point (other than point B).
Since we have shown that F lies on both semi-circles, then F must be this point of intersection of the two semi-circles.
Therefore, $DBEF$ is a square with side length 4, and F is the point of intersection of the two semi-circles.

Finally, we find the value of Y.
First we construct BF, the diagonal of square $DBEF$.
By symmetry, BF divides the shaded area Y into two equal areas.
Each of these equal areas, $\frac{Y}{2}$, is equal to the area of $\triangle BEF$ subtracted from the area of the quarter-circle BEF.
That is, $\frac{Y}{2} = \frac{1}{4}\pi(4)^2 - \frac{1}{2}(4)(4)$, and so $\frac{Y}{2} = 4\pi - 8$, or $Y = 8\pi - 16$.
The area of the shaded region is $X + Y = 2Y = 16\pi - 32$.
Of the answers given, $16\pi - 32$ is closest to 18.3.



ANSWER: (D)

25. *Solution 1*
Let the number of black plates, gold plates, and red plates be b, g and r, respectively (b, g and r are whole numbers).
Brady is stacking 600 plates, and so $b + g + r = 600$, where b is a multiple of 2, g is a multiple of 3, and r is a multiple of 6.
Rewrite this equation as $g = 600 - b - r$ and consider the right side of the equation.
Since 600 is a multiple of 2, and b is a multiple of 2, and r is a multiple of 2 (any multiple of 6 is a multiple of 2), then $600 - b - r$ is a multiple of 2 (the difference between even numbers is even).
Since the right side of the equation is a multiple of 2, then the left side, g, must also be a multiple of 2.
We are given that g is a multiple of 3, and since g is also a multiple of 2, then g must be an even multiple of 3 or a multiple of 6.

Similarly, rewriting the equation as $b = 600 - g - r$ and considering the right side of the equation: 600 is a multiple of 6, and g is a multiple of 6, and r is a multiple of 6, so then $600 - g - r$ is a multiple of 6 (the difference between multiples of 6 is a multiple of 6).
Since the right side of the equation is a multiple of 6, then the left side, b, must also be a multiple of 6.
That is, each of b, g and r is a multiple of 6, and so we let $b = 6B$, $g = 6G$, and $r = 6R$ where B, G and R are whole numbers.
So then the equation $b + g + r = 600$ becomes $6B + 6G + 6R = 600$, which is equivalent to $B + G + R = 100$ after dividing by 6.
Each solution to the equation $B + G + R = 100$ corresponds to a way that Brady could stack the 600 plates, and every possible way that Brady could stack the 600 plates corresponds to a solution to the equation $B + G + R = 100$.
For example, $B = 30, G = 50, R = 20$ corresponds to $b = 6 \times 30 = 180, g = 6 \times 50 = 300$, $r = 6 \times 20 = 120$, which corresponds to Brady stacking 180 black plates, below 300 gold plates, which are below 120 red plates.
Since $B + G + R = 100$, then each of B, G and R has a maximum possible value of 100.
If $B = 100$, then $G = R = 0$.
If $B = 99$, $G + R = 100 - B = 1$.
Thus, $G = 0$ and $R = 1$ or $G = 1$ and $R = 0$.
That is, once we assign values for B and G, then there is no choice for R since it is determined by the equation $R = 100 - B - G$.
Thus, to determine the number of solutions to the equation $B + G + R = 100$, we must determine the number of possible pairs (B, G) which lead to a solution.
For example, above we showed that the three pairs $(100, 0)$, $(99, 0)$, and $(99, 1)$ each correspond to a solution to the equation.
Continuing in this way, we determine all possible pairs (B, G) (which give R) that satisfy $B + G + R = 100$.

Value of B	Value of G	Number of plates: b, g, r
100	0	600,0,0
99	0	594,0,6
99	1	594,6,0
98	0	588,0,12
98	1	588,6,6
98	2	588,12,0
97	0	582,0,18
97	1	588,6,12
97	2	588,12,6
97	3	588,18,0
$\vdots$	$\vdots$	$\vdots$
$100 - n$	0	$6(100 - n), 0, 6n$
$100 - n$	1	$6(100 - n), 6, 6(n - 1)$
$100 - n$	2	$6(100 - n), 12, 6(n - 2)$
$\vdots$	$\vdots$	$\vdots$
$100 - n$	n	$6(100 - n), 6n, 0$

We see from the table that if the value of B is $100 - n$ for some whole number $n \leq 100$, then G can equal any whole number from 0 to n and so there are $n + 1$ possible choices for G.
That is, when $B = 100$, there is 1 choice for G, when $B = 99$, there are 2 choices for G, when

$B = 98$, there are 3 choices for G, and so on.
Each additional decrease of 1 in B gives 1 additional choice for G until we arrive at $B = 0$ ($n = 100$), which gives $n + 1 = 100 + 1 = 101$ possible choices for G $(0, 1, 2, 3, \ldots, 100)$.
Therefore, the total number of solutions to $B + G + R = 100$ is given by the sum $1 + 2 + 3 + \cdots + 99 + 100 + 101$.
Using the fact that the sum of the first m positive integers $1 + 2 + 3 + \cdots + m$ is equal to $\frac{m(m+1)}{2}$, we get $1 + 2 + 3 + \cdots + 99 + 100 + 101 = \frac{101(102)}{2} = 5151$.
Since each of these solutions corresponds to a way that Brady could stack the plates, there are 5151 ways that Brady could stack the plates under the given conditions.

Solution 2
In a given way of stacking the plates, let b be the number of groups of 2 black plates, g be the number of groups of 3 gold plates, and r be the number of groups of 6 red plates.
Then there are $2b$ black plates, $3g$ gold plates, and $6r$ red plates.
Since the total number of plates in a stack is 600, then $2b + 3g + 6r = 600$.
We note that the numbers of black, gold and red plates completely determines the stack (we cannot rearrange the plates in any way), and so the number of ways of stacking the plates is the same as the number of ways of solving the equation $2b + 3g + 6r = 600$ where b, g, r are integers that are greater than or equal to 0.
Since r is at least 0 and $6r$ is at most 600, then the possible values for r are $0, 1, 2, 3, \ldots, 98, 99, 100$.

When $r = 0$, we obtain $2b + 3g = 600$.
Since g is at least 0 and $3g$ is at most 600, then g is at most 200.
Since $2b$ and 600 are even, then $3g$ is even, so g is even.
Therefore, the possible values for g are $0, 2, 4, \ldots, 196, 198, 200$.
Since $200 = 100 \times 2$, then there are 101 possible values for g.
When $g = 0$, we get $2b = 600$ and so $b = 300$.
When $g = 2$, we get $2b = 600 - 3 \times 2 = 594$ and so $b = 297$.
Each time we increase g by 2, the number of gold plates increases by 6, so the number of black plates must decrease by 6, and so b decreases by 3.
Thus, as we continue to increase g by 2s from 2 to 200, the values of b will decrease by 3s from 297 to 0.
In other words, every even value for g does give an integer value for b.
Therefore, when $r = 0$, there are 101 solutions to the equation.

When $r = 1$, we obtain $2b + 3g = 600 - 6 \times 1 = 594$.
Again, g is at least 0, is even, and is at most $594 \div 3 = 198$.
Therefore, the possible values of g are $0, 2, 4, \ldots, 194, 196, 198$.
Again, each value of g gives a corresponding integer value of b.
Therefore, when $r = 1$, there are 100 solutions to the equation.

Consider the case of an unknown value of r, which gives $2b + 3g = 600 - 6r$.
Again, g is at least 0 and is even.
Also, the maximum possible value of g is $\frac{600 - 6r}{3} = 200 - 2r$.
This means that there are $(100 - r) + 1 = 101 - r$ possible values for g. (Can you see why?)
Again, each value of g gives a corresponding integer value of b.
Therefore, for a general r between 0 and 100, inclusive, there are $101 - r$ solutions to the equation.

We make a table to summarize the possibilities:

r	g	b	# of solutions
0	$0, 2, 4, \ldots, 196, 198, 200$	$300, 297, 294, \ldots, 6, 3, 0$	101
1	$0, 2, 4, \ldots, 194, 196, 198$	$297, 294, 291 \ldots, 6, 3, 0$	100
2	$0, 2, 4, \ldots, 192, 194, 196$	$294, 291, 288 \ldots, 6, 3, 0$	99
$\vdots$	$\vdots$	$\vdots$	$\vdots$
98	$0, 2, 4$	$6, 3, 0$	3
99	$0, 2$	$3, 0$	2
100	0	0	1

Therefore, the total number of ways of stacking the plates is

$$101 + 100 + 99 + \cdots + 3 + 2 + 1$$

We note that the integers from 1 to 100 can be grouped into 50 pairs each of which has a sum of 101 $(1 + 100, 2 + 99, 3 + 98, \ldots, 50 + 51)$.
Therefore, the number of ways that Brady could stack the plates is $101 + 50 \times 101 = 5151$.

ANSWER: (E)