The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
cemc.uwaterloo.ca

Gauss Contest

Grade 8

(The Grade 7 Contest is on the reverse side)

Wednesday, May 16, 2018
(in North America and South America)

Thursday, May 17, 2018
(outside of North America and South America)

UNIVERSITY OF
WATERLOO

Time: 1 hour

Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) previously stored information such as formulas, programs, notes, etc., (iv) a computer algebra system, (v) dynamic geometry software.

Instructions

1. Do not open the contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your answer sheet. If you are not sure, ask your teacher to explain it.
4. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. When you have made your choice, enter the appropriate letter for that question on your answer sheet.
5. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
6. Diagrams are *not* drawn to scale. They are intended as aids only.
7. When your supervisor instructs you to start, you will have *sixty* minutes of working time.

The name, school and location of some top-scoring students will be published on the Web site, cemc.uwaterloo.ca. You will also be able to find copies of past Contests and excellent resources for enrichment, problem solving and contest preparation.

Grade 8

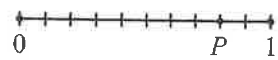
Scoring:	There is *no penalty* for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. The cost of 1 melon is \$3. What is the cost of 6 melons?

   (A) \$12 **(B)** \$15 **(C)** \$18 **(D)** \$21 **(E)** \$24

2. In the diagram, the number line is divided into 10 equal parts. The numbers 0, 1 and P are marked on the line. What is the value of P?



   (A) 0.2 **(B)** 0.6 **(C)** 0.7
   (D) 0.8 **(E)** 0.9

3. The value of $(2+3)^2 - (2^2+3^2)$ is

   (A) 50 **(B)** 12 **(C)** 15 **(D)** -15 **(E)** -12

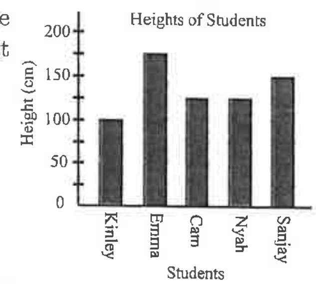
4. Lakshmi is travelling at 50 km/h. How many kilometres does she travel in 30 minutes?

   (A) 30 km **(B)** 50 km **(C)** 25 km **(D)** 150 km **(E)** 100 km

5. Evgeny has 3 roses, 2 tulips, 4 daisies, and 6 lilies. If he randomly chooses one of these flowers, what is the probability that it is a tulip?

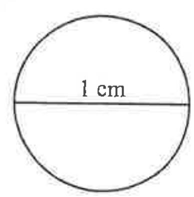
   (A) $\frac{3}{15}$ **(B)** $\frac{12}{15}$ **(C)** $\frac{6}{15}$ **(D)** $\frac{4}{15}$ **(E)** $\frac{2}{15}$

6. The heights of five students at Gleeson Middle School are shown in the graph. The range of the heights is closest to



   (A) 75 cm **(B)** 0 cm **(C)** 25 cm
   (D) 100 cm **(E)** 50 cm

7. The circle has a diameter of 1 cm, as shown. The circumference of the circle is between



   (A) 2 cm and 3 cm
   (B) 3 cm and 4 cm
   (C) 4 cm and 5 cm
   (D) 5 cm and 6 cm
   (E) 6 cm and 8 cm

8. Rich and Ben ate an entire chocolate cake. The ratio of the amount eaten by Rich to the amount eaten by Ben is 3 : 1. What percentage of the cake did Ben eat?

   (A) 66% **(B)** 50% **(C)** 75% **(D)** 25% **(E)** 10%

9. The 26 letters of the alphabet are written in order, clockwise around a circle. The *ciphertext* of a message is created by replacing each letter of the message by the letter that is 4 letters clockwise from the original letter. (This is called a *Caesar cipher*.) For example, the message ZAP has ciphertext DET. What is the ciphertext of the message WIN?

   (A) ALN **(B)** ZLN **(C)** AMR **(D)** AMQ **(E)** ZMQ

10. The sum of 3 consecutive even numbers is 312. What is the largest of these 3 numbers?
(A) 54 **(B)** 106 **(C)** 86 **(D)** 108 **(E)** 102

Part B: Each correct answer is worth 6.

11. If $4x + 12 = 48$, the value of x is
(A) 12 **(B)** 32 **(C)** 15 **(D)** 6 **(E)** 9

12. There is a 3 hour time difference between Vancouver and Toronto. For example, when it is 1:00 p.m. in Vancouver, it is 4:00 p.m. in Toronto. What time is it in Vancouver when it is 6:30 p.m. in Toronto?
(A) 9:30 p.m **(B)** 2:30 p.m. **(C)** 3:30 p.m. **(D)** 8:30 p.m. **(E)** 4:30 p.m.

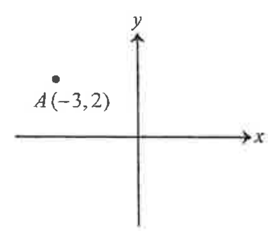
13. Mateo and Sydney win a contest. As his prize, Mateo receives \$20 every hour for one week. As her prize, Sydney receives \$400 every day for one week. What is the difference in the total amounts of money that they receive over the one week period?
(A) \$560 **(B)** \$80 **(C)** \$1120 **(D)** \$380 **(E)** \$784

14. The number 2018 has exactly two divisors that are prime numbers. The sum of these two prime numbers is
(A) 793 **(B)** 1011 **(C)** 38 **(D)** 209 **(E)** 507

15. Five classmates, Barry, Hwan, Daya, Cindy, and Ed will compete in a contest. There are no ties allowed. In how many ways can first, second and third place awards be given out?
(A) 6 **(B)** 60 **(C)** 125 **(D)** 3 **(E)** 27

16. There are several groups of six integers whose product is 1. Which of the following cannot be the sum of such a group of six integers?
(A) -6 **(B)** -2 **(C)** 0 **(D)** 2 **(E)** 6

17. A translation moves point $A(-3, 2)$ to the right 5 units and up 3 units. This translation is done a total of 6 times. After these translations, the point is at (x, y). What is the value of $x + y$?

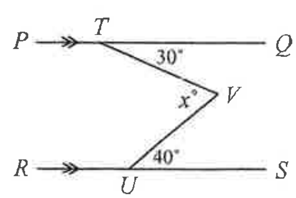


(A) 34 **(B)** 49 **(C)** 53
(D) 47 **(E)** 43

18. The volume of a rectangular prism is 30 cm^3. The length of the prism is doubled, the width is tripled, and the height is divided by four. The volume of the new prism is
(A) 31 cm^3 **(B)** 120 cm^3 **(C)** 60 cm^3 **(D)** 90 cm^3 **(E)** 45 cm^3

19. The mean (average) height of a group of children would be increased by 6 cm if 12 of the children in the group were each 8 cm taller. How many children are in the group?
(A) 16 **(B)** 14 **(C)** 21 **(D)** 26 **(E)** 9

20. Line segments PQ and RS are parallel. Points T, U, V are placed so that $\angle QTV = 30°$, $\angle SUV = 40°$, and $\angle TVU = x°$, as shown. What is the value of x?



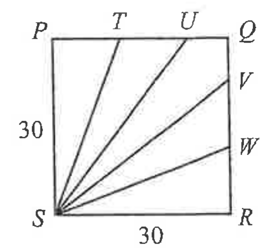
(A) 80 **(B)** 85 **(C)** 65
(D) 70 **(E)** 75

Part C: Each correct answer is worth 8.

21. A bag contains marbles of five different colours. One marble is chosen at random. The probability of choosing a brown marble is 0.3. Choosing a brown marble is three times as likely as choosing a purple marble. Choosing a green marble is equally likely as choosing a purple marble. Choosing a red marble is equally likely as choosing a yellow marble. The probability of choosing a marble that is either red or green is

(A) 0.2 **(B)** 0.25 **(C)** 0.35 **(D)** 0.4 **(E)** 0.55

22. Square $PQRS$ has side length 30, as shown. The square is divided into 5 regions of equal area: $\triangle SPT$, $\triangle STU$, $\triangle SVW$, $\triangle SWR$, and quadrilateral $SUQV$. The value of $\dfrac{SU}{ST}$ is closest to



(A) 1.17 **(B)** 1.19 **(C)** 1.21
(D) 1.23 **(E)** 1.25

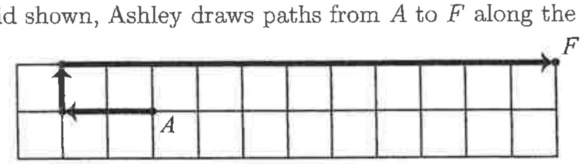
23. The smallest positive integer n for which $n(n+1)(n+2)$ is a multiple of 5 is $n = 3$. All positive integers, n, for which $n(n+1)(n+2)$ is a multiple of 5 are listed in increasing order. What is the 2018^{th} integer in the list?

(A) 3362 **(B)** 3360 **(C)** 3363 **(D)** 3361 **(E)** 3364

24. Lynne chooses four distinct digits from 1 to 9 and arranges them to form the 24 possible four-digit numbers. These 24 numbers are added together giving the result N. For all possible choices of the four distinct digits, what is the largest sum of the distinct prime factors of N?

(A) 157 **(B)** 148 **(C)** 127 **(D)** 146 **(E)** 124

25. In the 2×12 grid shown, Ashley draws paths from A to F along the gridlines.



In every path,

- there are two or more arrows arranged head to tail,
- the tail of the first arrow starts at A and the head of the last arrow ends at F,
- two consecutive arrows must be perpendicular to one another,
- no two arrows can intersect at more than one point, and
- all arrows have different lengths.

The path from A to F shown consists of arrows of three different lengths: left 2, up 1, right 11. How many different paths are there from A to F?

(A) 54 **(B)** 55 **(C)** 56 **(D)** 57 **(E)** 58

Grade 8

1. Since the cost of 1 melon is \$3, then the cost of 6 melons is $6 \times \$3 = \$18$.

ANSWER: (C)

2. The number line shown has length $1 - 0 = 1$.
The number line is divided into 10 equal parts, and so each part has length $1 \div 10 = 0.1$.
The P is positioned 2 of these equal parts before 1, and so the value of P is $1 - (2 \times 0.1) = 1 - 0.2 = 0.8$.
(Similarly, we could note that P is positioned 8 equal parts after 0, and so the value of P is $8 \times 0.1 = 0.8$.)

ANSWER: (D)

3. Following the correct order of operations, we get $(2+3)^2 - (2^2+3^2) = 5^2 - (4+9) = 25 - 13 = 12$.

ANSWER: (B)

4. Since Lakshmi is travelling at 50 km each hour, then in one half hour (30 minutes) she will travel $50 \div 2 = 25$ km.

ANSWER: (C)

5. Exactly 2 of the $3 + 2 + 4 + 6 = 15$ flowers are tulips.
Therefore, the probability that Evgeny randomly chooses a tulip is $\frac{2}{15}$.

ANSWER: (E)

6. The range of the students' heights is equal to the difference between the height of the tallest student and the height of the shortest student.
Reading from the graph, Emma is the tallest student and her height is approximately 175 cm.
Kinley is the shortest student and her height is approximately 100 cm.
Therefore, the range of heights is closest to $175 - 100 = 75$ cm.

ANSWER: (A)

7. *Solution 1*
The circumference of a circle, C, is given by the formula $C = \pi \times d$, where d is the diameter of the circle.
Since the circle has a diameter of 1 cm, then its circumference is $C = \pi \times 1 = \pi$ cm.
Since π is approximately 3.14, then the circumference of the circle is between 3 cm and 4 cm.

Solution 2
The circumference of a circle, C, is given by the formula $C = 2 \times \pi \times r$, where r is the radius of the circle.
Since the circle has a diameter of 1 cm, then its radius is $r = \frac{1}{2}$ cm, and so the circumference is $C = 2 \times \pi \times \frac{1}{2} = \pi$ cm.
Since π is approximately 3.14, then the circumference of the circle is between 3 cm and 4 cm.

ANSWER: (B)

8. The ratio of the amount of cake eaten by Rich to the amount of cake eaten by Ben is $3 : 1$.
Thus, if the cake was divided into 4 pieces of equal size, then Rich ate 3 pieces and Ben ate 1 piece or Ben ate $\frac{1}{4}$ of the cake.
Converting to a percent, Ben ate $\frac{1}{4} \times 100\% = 0.25 \times 100\% = 25\%$ of the cake.

ANSWER: (D)

9. Moving 3 letters clockwise from W, we arrive at the letter Z.
Moving 1 letter clockwise from the letter Z, the alphabet begins again at A.
Therefore, the letter that is 4 letters clockwise from W is A.
Moving 4 letters clockwise from I, we arrive at the letter M.
Moving 4 letters clockwise from N, we arrive at the letter R.
The ciphertext of the message WIN is AMR.

ANSWER: (C)

10. The smallest of 3 consecutive even numbers is 2 less than the middle number.
The largest of 3 consecutive even numbers is 2 more than the middle number.
Therefore, the sum of 3 consecutive even numbers is three times the middle number.
To see this, consider subtracting 2 from the largest of the 3 numbers, and adding 2 to the smallest of the 3 numbers.
Since we have subtracted 2 and also added 2, then the sum of these 3 numbers is equal to the sum of the original 3 numbers.
However, if we subtract 2 from the largest number, the result is equal to the middle number, and if we add 2 to the smallest number, the result is equal to the middle number.
Therefore, the sum of any 3 consecutive even numbers is equal to three times the middle number.
Since the sum of the 3 consecutive even numbers is 312, then the middle number is equal to $312 \div 3 = 104$.
If the middle number is 104, then the largest of the 3 consecutive even numbers is $104+2 = 106$.
(We may check that $102 + 104 + 106$ is indeed equal to 312.)

ANSWER: (B)

11. If $4x + 12 = 48$, then $4x = 48 - 12$ or $4x = 36$, and so $x = \frac{36}{4} = 9$.

ANSWER: (E)

12. The time in Vancouver is 3 hours earlier than the time in Toronto.
Therefore, when it is 6:30 p.m. in Toronto, the time in Vancouver is 3:30 p.m..

ANSWER: (C)

13. *Solution 1*
Mateo receives \$20 every hour for one week.
Since there are 24 hours in each day, and 7 days in each week, then Mateo receives $\$20 \times 24 \times 7 = \$3360$ over the one week period.
Sydney receives \$400 every day for one week.
Since there are 7 days in each week, then Sydney receives $\$400 \times 7 = \$2800$ over the one week period.
The difference in the total amounts of money that they receive over the one week period is $\$3360 - \$2800 = \$560$.

Solution 2
Mateo receives \$20 every hour for one week.
Since there are 24 hours in each day, then Mateo receives $\$20 \times 24 = \$480$ each day.
Sydney receives \$400 each day, and so Mateo receives $\$480 - \$400 = \$80$ more than Sydney each day.
Since there are 7 days in one week, then the difference in the total amounts of money that they receive over the one week period is $\$80 \times 7 = \$560$.

ANSWER: (A)

14. Since $2018 = 2 \times 1009$, and both 2 and 1009 are prime numbers, then the required sum is $2 + 1009 = 1011$.
Note: In the question, we are given that 2018 has exactly two divisors that are prime numbers, and since 2 is a prime divisor of 2018, then 1009 must be the other prime divisor.

ANSWER: (B)

15. The first place award can be given out to any one of the 5 classmates.
Once the first place award has been given, there are 4 classmates remaining who could be awarded second place (since the classmate who was awarded first place cannot also be awarded second place).
For each of the 5 possible first place winners, there are 4 classmates who could be awarded second place, and so there are 5×4 ways that the first and second place awards can be given out.
Once the first and second place awards have been given, there are 3 classmates remaining who could be awarded the third place award (since the classmates who were awarded first place and second place cannot also be awarded third place).
For each of the 5 possible first place winners, there are 4 classmates who could be awarded second place, and there are 3 classmates who could be awarded third place.
That is, there are $5 \times 4 \times 3 = 60$ ways that the first, second and third place awards can be given out.

ANSWER: (B)

16. For the product of two integers to equal 1, the two integers must both equal 1 or must both equal -1.
Similarly, if the product of six integers is equal to 1, then each of the six integers must equal 1 or -1.
For the product of six integers, each of which is equal to 1 or -1, to equal 1, the number of -1s must be even, because an odd number of -1s would give a product that is negative.
That is, there must be zero, two, four or six -1s among the six integers.
We summarize these four possibilities in the table below.

Number of -1s	Product of the six integers	Sum of the six integers
0	$(1)(1)(1)(1)(1)(1) = 1$	$1+1+1+1+1+1=6$
2	$(-1)(-1)(1)(1)(1)(1) = 1$	$(-1)+(-1)+1+1+1+1=2$
4	$(-1)(-1)(-1)(-1)(1)(1) = 1$	$(-1)+(-1)+(-1)+(-1)+1+1=-2$
6	$(-1)(-1)(-1)(-1)(-1)(-1) = 1$	$(-1)+(-1)+(-1)+(-1)+(-1)+(-1)=-6$

Of the answers given, the sum of such a group of six integers cannot equal 0.

ANSWER: (C)

17. *Solution 1*
Each translation to the right 5 units increases the x-coordinate of point A by 5.
Similarly, each translation up 3 units increases the y-coordinate of point A by 3.
After 1 translation, the point $A(-3,2)$ would be at $B(-3+5, 2+3)$ or $B(2,5)$.
After 2 translations, the point $A(-3,2)$ would be at $C(2+5, 5+3)$ or $C(7,8)$.
After 3 translations, the point $A(-3,2)$ would be at $D(7+5, 8+3)$ or $D(12,11)$.
After 4 translations, the point $A(-3,2)$ would be at $E(12+5, 11+3)$ or $E(17,14)$.
After 5 translations, the point $A(-3,2)$ would be at $F(17+5, 14+3)$ or $F(22,17)$.
After 6 translations, the point $A(-3,2)$ would be at $G(22+5, 17+3)$ or $G(27,20)$.
After these 6 translations, the point is at $(27,20)$ and so $x + y = 27 + 20 = 47$.

Solution 2
Each translation to the right 5 units increases the x-coordinate of point A by 5.
Similarly, each translation up 3 units increases the y-coordinate of point A by 3.
Therefore, each translation of point $A(-3,2)$ to the right 5 units and up 3 units increases the sum of the x- and y-coordinates, $x+y$, by $5+3=8$.
After 6 of these translations, the sum $x+y$ will increase by $6\times 8=48$.
The sum of the x- and y-coordinates of point $A(-3,2)$ is $-3+2=-1$.
After these 6 translations, the value of $x+y$ is $-1+48=47$.

ANSWER: (D)

18. *Solution 1*
The volume of any rectangular prism is given by the product of the length, the width, and the height of the prism.
When the length of the prism is doubled, the product of the new length, the width, and the height of the prism doubles, and so the volume of the prism doubles.
Since the original prism has a volume of 30 cm^3, then doubling the length creates a new prism with volume $30\times 2=60$ cm^3.
When the width of this new prism is tripled, the product of the length, the new width, and the height of the prism is tripled, and so the volume of the prism triples.
Since the prism has a volume of 60 cm^3, then tripling the width creates a new prism with volume $60\times 3=180$ cm^3.
When the height of the prism is divided by four, the product of the length, the width, and the new height of the prism is divided by four, and so the volume of the prism is divided by four.
Since the prism has a volume of 180 cm^3, then dividing the height by four creates a new prism with volume $180\div 4=45$ cm^3.

Solution 2
The volume of any rectangular prism is given by the product of its length, l, its width, w, and its height, h, which equals lwh.
When the length of the prism is doubled, the length of the new prism is $2l$.
Similarly, when the width is tripled, the new width is $3w$, and when the height is divided by four, the new height is $\frac{1}{4}h$.
Therefore, the volume of the new prism is the product of its length, $2l$, its width, $3w$, and its height, $\frac{1}{4}h$, which equals $(2l)(3w)(\frac{1}{4}h)$ or $\frac{3}{2}lwh$.
That is, the volume of the new prism is $\frac{3}{2}$ times larger than the volume of the original prism.
Since the original prism has a volume of 30 cm^3, then doubling the length, tripling the width, and dividing the height by four, creates a new prism with volume $30\times\frac{3}{2}=\frac{90}{2}=45$ cm^3.

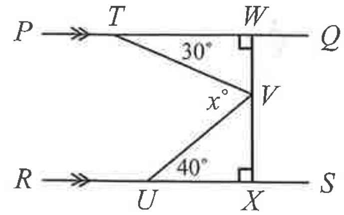
ANSWER: (E)

19. The mean height of the group of children is equal to the sum of the heights of the children divided by the number of children in the group.
Therefore, the mean height of the group of children increases by 6 cm if the sum of the increases in the heights of the children, divided by the number of children in the group, is equal to 6.
If 12 of the children were each 8 cm taller, then the sum of the increases in the heights of the children would be $12\times 8=96$ cm.
Thus, 96 divided by the number of children in the group is equal to 6.
Since $96\div 16=6$, then the number of children in the group is 16.

ANSWER: (A)

20. *Solution 1*

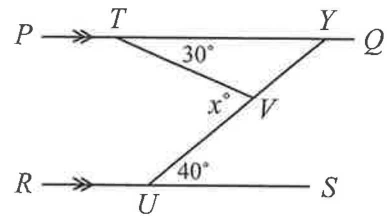
We begin by constructing a line segment WX perpendicular to PQ and passing through V, as shown.



Since RS is parallel to PQ, then WX is also perpendicular to RS.
In $\triangle TWV$, $\angle TWV = 90°$ and $\angle WTV = 30°$.
Since the sum of the angles in a triangle is $180°$, then $\angle TVW = 180° - 30° - 90° = 60°$.
In $\triangle UXV$, $\angle UXV = 90°$ and $\angle VUX = 40°$.
Similarly, $\angle UVX = 180° - 40° - 90° = 50°$.
Since WX is a straight line segment, then $\angle TVW + \angle TVU + \angle UVX = 180°$.
That is, $60° + \angle TVU + 50° = 180°$ or $\angle TVU = 180° - 60° - 50°$ or $\angle TVU = 70°$, and so $x = 70$.

Solution 2

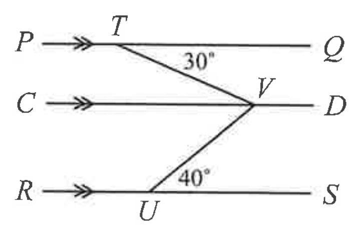
We begin by extending line segment UV to meet PQ at Y, as shown.



Since RS is parallel to PQ, then $\angle TYV$ and $\angle VUS$ are alternate angles, and so $\angle TYV = \angle VUS = 40°$.
In $\triangle TYV$, $\angle TYV = 40°$ and $\angle YTV = 30°$.
Since the sum of the angles in a triangle is $180°$, then $\angle TVY = 180° - 40° - 30° = 110°$.
Since UY is a straight line segment, then $\angle TVU + \angle TVY = 180°$.
That is, $\angle TVU + 110° = 180°$ or $\angle TVU = 180° - 110°$ or $\angle TVU = 70°$, and so $x = 70$.

Solution 3

We begin by constructing a line segment CD parallel to both PQ and RS, and passing through V, as shown.



Since CD is parallel to PQ, then $\angle QTV$ and $\angle TVC$ are alternate angles, and so $\angle TVC = \angle QTV = 30°$.
Similarly, since CD is parallel to RS, then $\angle CVU$ and $\angle VUS$ are alternate angles, and so $\angle CVU = \angle VUS = 40°$.
Since $\angle TVU = \angle TVC + \angle CVU$, then $\angle TVU = 30° + 40° = 70°$, and so $x = 70$.

ANSWER: (D)

21. *Solution 1*

We begin by assuming that there are 100 marbles in the bag.
The probability of choosing a brown marble is 0.3, and so the number of brown marbles in the bag must be 30 since $\frac{30}{100} = 0.3$.
Choosing a brown marble is three times as likely as choosing a purple marble, and so the number of purple marbles in the bag must be $30 \div 3 = 10$.
Choosing a green marble is equally likely as choosing a purple marble, and so there must also be 10 green marbles in the bag.
Since there are 30 brown marbles, 10 purple marbles, and 10 green marbles in the bag, then there are $100 - 30 - 10 - 10 = 50$ marbles in the bag that are either red or yellow.
Choosing a red marble is equally likely as choosing a yellow marble, and so the number of red marbles in the bag must equal the number of yellow marbles in the bag.
Therefore, the number of red marbles in the bag is $50 \div 2 = 25$.
Of the 100 marbles in the bag, there are $25 + 10 = 35$ marbles that are either red or green.
The probability of choosing a marble that is either red or green is $\frac{35}{100} = 0.35$.

Solution 2

The probability of choosing a brown marble is 0.3.
The probability of choosing a brown marble is three times that of choosing a purple marble, and so the probability of choosing a purple marble is $0.3 \div 3 = 0.1$.
The probability of choosing a green marble is equal to that of choosing a purple marble, and so the probability of choosing a green marble is also 0.1.
Let the probability of choosing a red marble be p.
The probability of choosing a red marble is equal to that of choosing a yellow marble, and so the probability of choosing a yellow marble is also p.
The total of the probabilities of choosing a marble must be 1.
Therefore, $0.3 + 0.1 + 0.1 + p + p = 1$ or $0.5 + 2p = 1$ or $2p = 0.5$, and so $p = 0.5 \div 2 = 0.25$.
The probability of choosing a red marble is 0.25 and the probability of choosing a green marble is 0.1, and so the probability of choosing a marble that is either red or green is $0.25+0.1 = 0.35$.

ANSWER: (C)

22. The area of square $PQRS$ is $(30)(30) = 900$.
Each of the 5 regions has equal area, and so the area of each region is $900 \div 5 = 180$.
The area of $\triangle SPT$ is equal to $\frac{1}{2}(PS)(PT) = \frac{1}{2}(30)(PT) = 15(PT)$.
The area of $\triangle SPT$ is 180, and so $15(PT) = 180$ or $PT = 180 \div 15 = 12$.
The area of $\triangle STU$ is 180.
Let the base of $\triangle STU$ be UT.
The height of $\triangle STU$ is equal to PS since PS is the perpendicular distance between base UT (extended) and the vertex S.
The area of $\triangle STU$ is equal to $\frac{1}{2}(PS)(UT) = \frac{1}{2}(30)(UT) = 15(UT)$.
The area of $\triangle STU$ is 180, and so $15(UT) = 180$ or $UT = 180 \div 15 = 12$.
In $\triangle SPT$, $\angle SPT = 90°$. By the Pythagorean Theorem, $ST^2 = PS^2 + PT^2$ or $ST^2 = 30^2 + 12^2$ or $ST^2 = 900 + 144 = 1044$, and so $ST = \sqrt{1044}$ (since $ST > 0$).
In $\triangle SPU$, $\angle SPU = 90°$ and $PU = PT + UT = 12 + 12 = 24$.
By the Pythagorean Theorem, $SU^2 = PS^2 + PU^2$ or $ST^2 = 30^2 + 24^2$ or $SU^2 = 900 + 576 = 1476$, and so $SU = \sqrt{1476}$ (since $SU > 0$).
Therefore, $\dfrac{SU}{ST} = \dfrac{\sqrt{1476}}{\sqrt{1044}}$ which is approximately equal to 1.189.
Of the answers given, $\dfrac{SU}{ST}$ is closest to 1.19.

ANSWER: (B)

23. *Solution 1*
In the table, we determine the value of the product $n(n+1)(n+2)$ for the first 10 positive integers:

n	$n(n+1)(n+2)$
1	$1 \times 2 \times 3 = 6$
2	$2 \times 3 \times 4 = 24$
3	$3 \times 4 \times 5 = 60$
4	$4 \times 5 \times 6 = 120$
5	$5 \times 6 \times 7 = 210$
6	$6 \times 7 \times 8 = 336$
7	$7 \times 8 \times 9 = 504$
8	$8 \times 9 \times 10 = 720$
9	$9 \times 10 \times 11 = 990$
10	$10 \times 11 \times 12 = 1320$

From the table, we see that $n(n+1)(n+2)$ is a multiple of 5 when $n = 3, 4, 5, 8, 9, 10$.
In general, because 5 is a prime number, the product $n(n+1)(n+2)$ is a multiple of 5 exactly when at least one of its factors $n, n+1, n+2$ is a multiple of 5.
A positive integer is a multiple of 5 when its units (ones) digit is either 0 or 5.
Next, we make a table that lists the units digits of $n+1$ and $n+2$ depending on the units digit of n:

Units digit of n	Units digit of $n+1$	Units digit of $n+2$
1	2	3
2	3	4
3	4	5
4	5	6
5	6	7
6	7	8
7	8	9
8	9	0
9	0	1
0	1	2

From the table, one of the three factors has a units digit of 0 or 5 exactly when the units digit of n is one of $3, 4, 5, 8, 9, 0$. (Notice that this agrees with the first table above.)
This means that 6 out of each block of 10 values of n ending at a multiple of 10 give a value for $n(n+1)(n+2)$ that is a multiple of 5.
We are asked for the 2018^{th} positive integer n for which $n(n+1)(n+2)$ is a multiple of 5.
Note that $2018 = 336 \times 6 + 2$.
This means that, in the first $336 \times 10 = 3360$ positive integers, there are $336 \times 6 = 2016$ integers n for which $n(n+1)(n+2)$ is a multiple of 5. (Six out of every ten integers have this property.)
We need to count two more integers along the list.
The next two integers n for which $n(n+1)(n+2)$ is a multiple of 5 will have units digits 3 and 4, and so are 3363 and 3364.
This means that 3364 is the 2018^{th} integer with this property.

Solution 2
In the table below, we determine the value of the product $n(n+1)(n+2)$ for the first 10 positive integers n.

n	$n(n+1)(n+2)$
1	$1 \times 2 \times 3 = 6$
2	$2 \times 3 \times 4 = 24$
3	$3 \times 4 \times 5 = 60$
4	$4 \times 5 \times 6 = 120$
5	$5 \times 6 \times 7 = 210$
6	$6 \times 7 \times 8 = 336$
7	$7 \times 8 \times 9 = 504$
8	$8 \times 9 \times 10 = 720$
9	$9 \times 10 \times 11 = 990$
10	$10 \times 11 \times 12 = 1320$

From the table, we see that the value of $n(n+1)(n+2)$ is not a multiple of 5 when $n = 1$ or when $n = 2$, but that $n(n+1)(n+2)$ is a multiple of 5 when $n = 3, 4, 5$.

Similarly, we see that the value of $n(n+1)(n+2)$ is a not multiple of 5 when $n = 6, 7$, but that $n(n+1)(n+2)$ is a multiple of 5 when $n = 8, 9, 10$.
That is, if we consider groups of 5 consecutive integers beginning at $n = 1$, it appears that for the first 2 integers in the group, the value of $n(n+1)(n+2)$ is not a multiple of 5, and for the last 3 integers in the group, the value of $n(n+1)(n+2)$ is a multiple of 5.
Will this pattern continue?
Since 5 is a prime number, then for each value of $n(n+1)(n+2)$ that is a multiple of 5, at least one of the factors $n, n+1$ or $n+2$ must be divisible by 5.
(We also note that for each value of $n(n+1)(n+2)$ that is not a multiple of 5, each of n, $n+1$ and $n+2$ is not divisible by 5.)
For what values of n is at least one of n, $n+1$ or $n+2$ divisible by 5, and thus $n(n+1)(n+2)$ divisible by 5?
When n is a multiple of 5, then the value of $n(n+1)(n+2)$ is divisible by 5.
When n is one 1 less than a multiple of 5, then $n+1$ is a multiple of 5 and and so $n(n+1)(n+2)$ is divisible by 5.
Finally, when n is 2 less than a multiple of 5, then $n+2$ is a multiple of 5 and and so $n(n+1)(n+2)$ is divisible by 5.
We also note that when n is 3 less than a multiple of 5, each of n, $n+1$ (which is 2 less than a multiple of 5), and $n+2$ (which is 1 less than a multiple of 5) is not divisible by 5, and so $n(n+1)(n+2)$ is not divisible by 5.
Similarly, when n is 4 less than a multiple of 5, then each of n, $n+1$ (which is 3 less than a multiple of 5), and $n+2$ (which is 2 less than a multiple of 5) is not divisible by 5, and so $n(n+1)(n+2)$ is not divisible by 5.
We have shown that the value of $n(n+1)(n+2)$ is a multiple of 5 when n is: a multiple of 5, or 1 less than a multiple of 5, or 2 less than a multiple of 5.
We have also shown that the value of $n(n+1)(n+2)$ is not a multiple of 5 when n is: 3 less than a multiple of 5, or 4 less than a multiple of 5.
Since every positive integer is either a multiple of 5, or 1, 2, 3, or 4 less than a multiple of 5, we have considered the value of $n(n+1)(n+2)$ for all positive integers n.
In the first group of 5 positive integers from 1 to 5, there are exactly 3 integers n ($n = 3, 4, 5$) for which $n(n+1)(n+2)$ is a multiple of 5.
Similarly, in the second group of 5 positive integers from 6 to 10, there are exactly 3 integers n ($n = 8, 9, 10$) for which $n(n+1)(n+2)$ is a multiple of 5.
As was shown above, this pattern continues giving 3 values for n for which $n(n+1)(n+2)$ is a multiple of 5 in each successive group of 5 consecutive integers.
When these positive integers, n, are listed in increasing order, we are required to find the 2018^{th} integer in the list.
Since $2018 = 3 \times 672 + 2$, then among the first 672 successive groups of 5 consecutive integers (which is the first $5 \times 672 = 3360$ positive integers), there are exactly $3 \times 672 = 2016$ integers n for which $n(n+1)(n+2)$ is a multiple of 5.
The next two integers, 3361 and 3362, do not give values for n for which $n(n+1)(n+2)$ is a multiple of 5 (since 3361 is 4 less than a multiple of 5 and 3362 is 3 less than a multiple of 5).
The next two integers, 3363 and 3364, do give values for n for which $n(n+1)(n+2)$ is a multiple of 5.
Therefore, the 2018^{th} integer in the list is 3364.

ANSWER: (E)

Similarly, we see that the value of $n(n+1)(n+2)$ is a not multiple of 5 when $n = 6, 7$, but that $n(n+1)(n+2)$ is a multiple of 5 when $n = 8, 9, 10$.
That is, if we consider groups of 5 consecutive integers beginning at $n = 1$, it appears that for the first 2 integers in the group, the value of $n(n+1)(n+2)$ is not a multiple of 5, and for the last 3 integers in the group, the value of $n(n+1)(n+2)$ is a multiple of 5.
Will this pattern continue?
Since 5 is a prime number, then for each value of $n(n+1)(n+2)$ that is a multiple of 5, at least one of the factors $n, n+1$ or $n+2$ must be divisible by 5.
(We also note that for each value of $n(n+1)(n+2)$ that is not a multiple of 5, each of n, $n+1$ and $n+2$ is not divisible by 5.)
For what values of n is at least one of n, $n+1$ or $n+2$ divisible by 5, and thus $n(n+1)(n+2)$ divisible by 5?
When n is a multiple of 5, then the value of $n(n+1)(n+2)$ is divisible by 5.
When n is one 1 less than a multiple of 5, then $n+1$ is a multiple of 5 and and so $n(n+1)(n+2)$ is divisible by 5.
Finally, when n is 2 less than a multiple of 5, then $n+2$ is a multiple of 5 and and so $n(n+1)(n+2)$ is divisible by 5.
We also note that when n is 3 less than a multiple of 5, each of n, $n+1$ (which is 2 less than a multiple of 5), and $n+2$ (which is 1 less than a multiple of 5) is not divisible by 5, and so $n(n+1)(n+2)$ is not divisible by 5.
Similarly, when n is 4 less than a multiple of 5, then each of n, $n+1$ (which is 3 less than a multiple of 5), and $n+2$ (which is 2 less than a multiple of 5) is not divisible by 5, and so $n(n+1)(n+2)$ is not divisible by 5.
We have shown that the value of $n(n+1)(n+2)$ is a multiple of 5 when n is: a multiple of 5, or 1 less than a multiple of 5, or 2 less than a multiple of 5.
We have also shown that the value of $n(n+1)(n+2)$ is not a multiple of 5 when n is: 3 less than a multiple of 5, or 4 less than a multiple of 5.
Since every positive integer is either a multiple of 5, or 1, 2, 3, or 4 less than a multiple of 5, we have considered the value of $n(n+1)(n+2)$ for all positive integers n.
In the first group of 5 positive integers from 1 to 5, there are exactly 3 integers n ($n = 3, 4, 5$) for which $n(n+1)(n+2)$ is a multiple of 5.
Similarly, in the second group of 5 positive integers from 6 to 10, there are exactly 3 integers n ($n = 8, 9, 10$) for which $n(n+1)(n+2)$ is a multiple of 5.
As was shown above, this pattern continues giving 3 values for n for which $n(n+1)(n+2)$ is a multiple of 5 in each successive group of 5 consecutive integers.
When these positive integers, n, are listed in increasing order, we are required to find the 2018^{th} integer in the list.
Since $2018 = 3 \times 672 + 2$, then among the first 672 successive groups of 5 consecutive integers (which is the first $5 \times 672 = 3360$ positive integers), there are exactly $3 \times 672 = 2016$ integers n for which $n(n+1)(n+2)$ is a multiple of 5.
The next two integers, 3361 and 3362, do not give values for n for which $n(n+1)(n+2)$ is a multiple of 5 (since 3361 is 4 less than a multiple of 5 and 3362 is 3 less than a multiple of 5).
The next two integers, 3363 and 3364, do give values for n for which $n(n+1)(n+2)$ is a multiple of 5.
Therefore, the 2018^{th} integer in the list is 3364.

ANSWER: (E)

25. Since the grid has height 2, then there are only two possible lengths for vertical arrows: 1 or 2. Since all arrows in any path have different lengths, then there can be at most 2 vertical arrows in any path.
This means that there cannot be more than 3 horizontal arrows in any path. (If there were 4 or more horizontal arrows then there would have to be 2 consecutive horizontal arrows in the path, which is forbidden by the requirement that two consecutive arrows must be perpendicular.)
This means that any path consists of at most 5 arrows.

Using the restriction that all arrows in any path must have different lengths, we now determine the possible combinations of lengths of vertical arrows and of horizontal arrows to get from A to F.
Once we have determined the possible combinations of vertical and horizontal arrows independently, we then try to combine and arrange them.
First, we look at vertical arrows.
The grid has height 2, and A is 1 unit below F so any combination of vertical arrows in a path must have a net results of 1 unit up.
We use "U" for up and "D" for down.
The possible combinations are:

U1 (up arrow with length 1)
D1, U2 (down arrow with length 1, up arrow with length 2)

Next, we look at horizontal arrows.
The grid has width 12, and A is 9 units to the left of F so any combination of horizontal arrows in a path must have a net result of 9 units right.
We use "R" for right and "L" for left.
Many of these combinations of arrows can be re-arranged in different orders. We will deal with this later.
We proceed by looking at combinations of 1 arrow, then 2 arrows, then 3 arrows.
We note that every combination of vertical arrows includes an arrow with length 1 so we can ignore any horizontal combination that uses an arrow of length 1.
Also, any combination of 3 horizontal arrows must be combined with a combination of 2 horizontal arrows, which have lengths 1 and 2.
Thus, we can ignore any combination of 3 horizontal arrows that includes either or both of an arrow of length 1 and length 2.

a) R9
b) R2, R7
c) R3, R6
d) R4, R5
e) L2, R11
f) L3, R12
g) L3, R4, R8
h) L3, R5, R7
i) L4, R3, R10
j) L4, R5, R8
k) L4, R6, R7
l) L5, R3, R11
m) L5, R4, R10
n) L5, R6, R8
o) L6, R3, R12
p) L6, R4, R11
q) L6, R5, R10
r) L6, R7, R8
s) L7, R4, R12
t) L7, R5, R11
u) L7, R6, R10
v) L8, R5, R12
w) L8, R6, R11
x) L8, R7, R10
y) L9, R6, R12
z) L9, R7, R11
aa) L9, R8, R10
ab) L10, R7, R12
ac) L10, R8, R11
ad) L11, R8, R12

There is only one combination of 1 horizontal arrow.
The combinations of 2 horizontal arrows are listed by including those with two right arrows first (in increasing order of length) and then those with left and right arrows (in increasing order of length).
The combinations of 3 arrows are harder to list completely.
There are no useful combinations that include either 3 right arrows or 2 left arrows, since in either case an arrow of length 1 or 2 would be required.
Here, we have listed combinations of 2 right arrows, then those with "L3" (left arrow of length 3), then those with "L4", and so on.

Now we combine the vertical and horizontal combinations to get the paths.
Each combination of arrow directions and lengths can be drawn to form a path.
Vertical combination U1 can only be combined with horizontal paths a through f, since it cannot be combined with 3 horizontal arrows.

a) There are 2 paths: U1/R9 or R9/U1.
b) There are 2 paths: R2/U1/R7 or R7/U1/R2.
c) Again, there are 2 paths.
d) Again, there are 2 paths.
e) There is 1 path: L2/U1/R11. This is because the arrows must alternate horizontal, vertical, horizontal and we cannot end with a left arrow.
f) Again, there is 1 path.

This is 10 paths so far.
Vertical combination D1, U2 can be combined with horizontal paths of lengths 1, 2 or 3.

a) There is 1 path: D1/R9/U2. This is because we cannot end with a down arrow.
b) Not possible because this would include two arrows of length 2.
c) There are 4 paths: R3/D1/R6/U2, R6/D1/R3/U2, D1/R3/U2/R6, D1/R6/U2/R3. We can interchange R3 and R6 as well as picking whether to start with a vertical or horizontal arrow.
d) Again, there are 4 paths.
e) Not possible because this would include two arrows of length 2.
f) There are 2 paths: L3/D1/R12/U2 and D1/L3/U2/R12.
g) There are 4 paths: L3/D1/R4/U2/R8, R4/D1/L3/U2/R8, L3/D1/R8/U2/R4, R8/D1/L3/U2/R4. Each such combination must start with a horizontal arrow, must end with a right arrow, and must have the down arrow before the up arrow.
h) Again, there are 4 paths.
i) There is 1 path: R3/D1/L4/U2/R10. We cannot begin with R10 or L4 since either would take the path off of the grid, and we must end with an arrow to the right.

j,k) There are 2 paths in each case. For example, with j we have R5/D1/L4/U2/R8 and R8/D1/L4/U2/R5.

l,m) As with i, there is 1 path in each case.

n) As with j, there are 2 paths.

o,p,q) As with i, there is 1 path in each case.

r) As with j, there are 2 paths.

s) to ad) In each of these 12 cases, there is 1 path as with i.

Including the previously counted 10 paths that use U1 only, we have

$$10 + 1 + 4 + 4 + 2 + 4 + 4 + 1 + 2(2) + 2(1) + 2 + 3(1) + 2 + 12(1) = 55$$

paths in total.

ANSWER: (B)